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Strong convergence results for random Jungck-Ishikawa and Jungck-Noor iterative schemes in Banach spaces

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Abstract

In this paper, we introduce a new random Jungck-Ishikawa and Jungck-Noor iterative schemes and discuss the strong convergence of them to a unique common random fixed point for two nonself random mappings under a general contractive condition in separable Banach spaces. Our results generalize and extend many results in this direction.

Keywords: random Jungck-Ishikawa iteration, random Jungck-Noor iteration, separable Banach spaces, monotone increasing function.
2010 MSC: 47H06, 54H25.

1. Introduction and Preliminaries

Random fixed point theory has been a wonderful branch of stochastic functional analysis with vast applicability. There are many new questions of measurability, probabilistic and statistical aspects of random solutions answered by the introduction of randomness. It is known that random fixed point theorems are stochastic generalization of classical fixed point theorems or deterministic fixed point theorems. Many authors are impressed by random fixed point theory especially, when Bharucha-Reid [2, 3] presented his paper which lead to the development of this theory. Approximation of fixed points was studied by several authors in deterministic fixed point theory [15–17]. A parallel development in random fixed point theory have attracted much attention during the last few years due to its increasing role in mathematics and applied sciences, some references are noted in [18, 24–27].

Recently, several general iterative schemes have been successfully applied to fixed point problems of operators and also for obtaining solutions of operator equations. The development of random fixed point iterations was initiated by Choudhury [6–11], where random Ishikawa iteration scheme was defined and its

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strong convergence to a random fixed point in Hilbert spaces was discussed.
In 2005, Singh et al. [28] proved the stability of Jungck type iterative procedure as follows:

Definition 1.1. (**Jungck-Mann Iteration Process**)
Let $(X, \|.\|)$ be a normed linear space and Y be an arbitrary set $S, T : Y \to X$ such that $T(Y) \subseteq S(Y)$, then for $x_\circ \in Y$, the sequence $\{Sx_n\}_{n=0}^{\infty}$ defined by

$$Sx_{n+1} = (1 - \alpha_n)Sx_n + \alpha_n Tx_n, \ n \geq 0, \tag{1.1}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a sequence of real numbers in $[0, 1]$.

Remark 1.2. If we put $S = I$ (where I is the identity mapping), $Y = X$, then we obtain Mann iteration process [19].

Many converges results in this direction are also proved by many authors (see [1, 4, 13, 22]). In 2008, great work published by Olatinwo and Imoru [23] which shows the convergence results of Jungck-Ishikawa iteration as the following:

Definition 1.3. (**Jungck-Ishikawa Iteration Process**)
Let $(X, \|.\|)$ be a Banach space and Y be an arbitrary set. Let $S, T : Y \to X$ be a nonself mappings such that $T(Y) \subseteq S(Y)$, $S(Y)$ is a complete subspace of X and S is injective, then for $x_\circ \in Y$, define the sequence $\{Sx_n\}_{n=0}^{\infty}$ iteratively by

$$\begin{cases} Sx_{n+1} = (1 - \alpha_n)Sx_n + \alpha_n Ty_n \\ Sy_n = (1 - \beta_n)Sx_n + \beta_n Tx_n \end{cases}, \tag{1.2}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are sequences in $[0, 1]$.

Remark 1.4. (i) If we take $S = I$, $Y = X$ in (1.2), we have the Ishikawa iteration process [14].
(ii) Taking $\beta_n = 0$ in (1.2), we get Jungck-Mann process (1.1).

The convergence results using Jungck-Noor three step iteration scheme were introduced by Olatinwo [21] as follows:

Definition 1.5. (**Jungck-Noor Iteration Process**)
Let $(X, \|.\|)$ be a Banach space and Y be an arbitrary set. Let $S, T : Y \to X$ be a nonself mappings such that $T(Y) \subseteq S(Y)$, $S(Y)$ is a complete subspace of X and S is injective, then for $x_\circ \in Y$, define the sequence $\{Sx_n\}_{n=0}^{\infty}$ iteratively by

$$\begin{cases} Sx_{n+1} = (1 - \alpha_n)Sx_n + \alpha_n Tz_n, \\ Sz_n = (1 - \beta_n)Sx_n + \beta_n Ty_n, \\ Sy_n = (1 - \gamma_n)Sx_n + \gamma_n Tx_n, \end{cases} \tag{1.3}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are sequences in $[0, 1]$.

Remark 1.6. (i) If we take $S = I$ and $Y = X$ in (1.3), we obtain Noor-three iteration process [20].
(ii) The iteration process (1.1) and (1.2) are special cases of iteration (1.3).

Throughout this paper, we assume that (Ω, Σ) is measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω, X stands for a separable Banach space, C is a nonempty closed convex subset of X. A function $T : \Omega \longrightarrow C$ is said to be measurable if $T^{-1}(B \cap C) \in \Sigma$ for each Borel subset B of E. A function $T : \Omega \times C \longrightarrow C$ is called a random operator, if $T(., x) : \Omega \longrightarrow C$ is measurable for every $x \in C$. A measurable function $\xi : \Omega \longrightarrow C$ is called a random fixed point for the operator $T : \Omega \times C \longrightarrow C$ if $T(\omega, \xi(\omega)) = \xi(\omega)$ for all $\omega \in \Omega$. A random operator $T : \Omega \times C \longrightarrow C$ is said to be continuous for any given $\omega \in \Omega$, $T(\omega, .) : \Omega \longrightarrow C$ is continuous.
In 2015, A newly defined random Jungck-Mann type iterative process was stated by Chandelkar et al. [5], who discussed the convergence of random Jungck-Mann iteration scheme to a common random fixed point under suitable contractive condition as the following:

Definition 1.7. (**Random Jungck-Mann Iteration**)
Let X be a separable Banach space and $S, T : \Omega \times C \to X$ be two nonself random mappings defined on a nonempty closed convex subset C of X, then the sequence $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ defined in the following: Let $x_\circ : \Omega \to C$ be an arbitrary measurable mapping for $\omega \in \Omega$, $n = 0, 1, ..$ with $T(\omega, C) \subseteq S(\omega, C)$,

$$S(\omega, x_{n+1}(\omega)) = (1 - \alpha_n)S(\omega, x_n(\omega)) + \alpha_n T(\omega, x_n(\omega)), \tag{1.4}$$

where $0 < \alpha_n < 1$, $n = 0, 1, 2, ..$ and $0 < \lim_{n\to\infty} \alpha_n = h < 1$.

Motivated by the above works, the main aim of this paper, is to introduce a new random Jungck-Ishikawa and random Jungck-Noor three step iterative schemes to establish the strong convergence to a common random fixed point using a general contractive condition for nonself random mappings in separable Banach spaces.

Now, we introduce random iteration schemes as follows:

Definition 1.8. (**Random Jungck-Ishikawa Iteration**)
Let $S, T : \Omega \times C \to X$ be two nonself random mappings defined on a nonempty closed convex subset C of X, then the sequence $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ defined as: Let $x_\circ : \Omega \to C$ be an arbitrary measurable mapping for $\omega \in \Omega$, $n = 0, 1, ..$ with $T(\omega, C) \subseteq S(\omega, C)$,

$$\begin{cases} S(\omega, x_{n+1}(\omega)) = (1 - \alpha_n)S(\omega, x_n(\omega)) + \alpha_n T(\omega, y_n(\omega)), \\ S(\omega, y_n(\omega)) = (1 - \beta_n)S(\omega, x_n(\omega)) + \beta_n T(\omega, x_n(\omega)), \end{cases} \tag{1.5}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are measurable sequences in $[0, 1]$, such that

(i) $0 < \alpha_n, \beta_n < 1 \ \forall n > 0$,

(ii) $\lim_{n\to\infty} \beta_n = 0$,

(iii) $\sum \alpha_n \beta_n = \infty$.

Definition 1.9. (**Random Jungck-Noor Iteration**)
Let $S, T : \Omega \times C \to X$ be two nonself random mappings defined on a nonempty closed convex subset C of X, then the sequence $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ defined as: Let $x_\circ : \Omega \to C$ be an arbitrary measurable mapping for $\omega \in \Omega$, $n = 0, 1, ..$ with $T(\omega, C) \subseteq S(\omega, C)$,

$$\begin{cases} S(\omega, x_{n+1}(\omega)) = (1 - \alpha_n)S(\omega, x_n(\omega)) + \alpha_n T(\omega, z_n(\omega)) \\ S(\omega, z_n(\omega)) = (1 - \beta_n)S(\omega, x_n(\omega)) + \beta_n T(\omega, y_n(\omega)) \\ S(\omega, y_n(\omega)) = (1 - \gamma_n)S(\omega, x_n(\omega)) + \gamma_n T(\omega, x_n(\omega)) \end{cases}, \tag{1.6}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are measurable sequences in $[0, 1]$, such that

(i) $0 < \alpha_n, \beta_n, \gamma_n < 1 \ \forall n > 0$,

(ii) $\lim_{n\to\infty} \beta_n = 0$,

(iii) $\sum \alpha_n \beta_n \gamma_n = \infty$.

Remark 1.10. If we take Ω is a singleton in (1.4)-(1.6), we get the nonrandom cases defined in (1.1)-(1.3), respectively.

The following contractive condition is a stochastic form of (Definition 1, [21]).

Definition 1.11. Let $S, T : \Omega \times C \to X$ be two nonself random mappings defined on a nonempty closed convex subset C of X. Consider $\varphi : R^+ \to R^+$ is a monotone increasing function with $\varphi(0) = 0$ and there exist real numbers $M \geq 0$ and $a \in [0, 1)$ such that for all $x, y \in C$, $T(\omega, C) \subseteq S(\omega, C)$, we have

$$\|T(\omega, x) - T(\omega, y)\| \leq \frac{\varphi(\|S(\omega, x) - T(\omega, x)\|) + a \|S(\omega, x) - S(\omega, y)\|}{1 + M \|S(\omega, x) - T(\omega, x)\|}. \tag{1.7}$$

2. Main Result

Now, we announce our results.

Theorem 2.1. *Let X be a separable Banach space, C be a nonempty closed convex subset of X. Let $T, S : \Omega \times C \to X$ be two non self continuous random operators defined on C with $T(\omega, C) \subseteq S(\omega, C)$ satisfying (1.7). Assume that random operators S and T have a random coincidence point. If the sequence $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ defined by (1.5) converges, then it converges to a unique common random fixed point of S and T.*

Proof. Suppose that the sequence $\{S(\omega, x_n(\omega))\}$ has a pointwise limit, that is, $\lim_{n\to\infty} S(\omega, x_n(\omega)) = p(\omega)$ for all $\omega \in \Omega$, since X be a separable Banach space, then the mapping $p(\omega) = S(\omega, f(\omega))$ is measurable mapping for any random operator $S : \Omega \times C \longrightarrow C$ and any measurable mapping $f : \Omega \longrightarrow C$ [12]. Therefore the sequence $\{S(\omega, x_n(\omega))\}$ constructed by the random Jungck-Ishikawa iteration (1.5) is a sequence of measurable mapping. Since $p(\omega)$ is measurable and C is convex, then $p : \Omega \to C$ being limit of measurable mapping sequence is also measurable. Since S and T have a random coincidence point i.e.

$$S(\omega, x(\omega)) = T(\omega, x(\omega)) = p(\omega), \tag{2.1}$$

for every $x(\omega) \in C$ be a measurable mapping.
For the strong convergence result, we have from (1.5) for $\omega \in \Omega$,

$$\begin{aligned} \|S(\omega, x_{n+1}(\omega)) - p(\omega)\| &= \|(1-\alpha_n)S(\omega, x_n(\omega)) + \alpha_n T(\omega, y_n(\omega)) - p(\omega)\| \\ &= \|(1-\alpha_n)(S(\omega, x_n(\omega)) - p(\omega)) + \alpha_n(T(\omega, y_n(\omega)) - p(\omega))\| \\ &\le (1-\alpha_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \alpha_n \|T(\omega, y_n(\omega)) - p(\omega)\|, \end{aligned}$$

using (2.1) and (1.7), we get

$$\begin{aligned} \|S(\omega, x_{n+1}(\omega)) - p(\omega)\| &\le (1-\alpha_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \alpha_n \|T(\omega, y_n(\omega)) - T(\omega, x(\omega))\| \\ &\le (1-\alpha_n)\|S(\omega, x_n(\omega)) - p(\omega)\| \\ &\quad + \alpha_n \left[\frac{\varphi(\|S(\omega, x(\omega)) - T(\omega, x(\omega))\| + a\|S(\omega, x(\omega)) - S(\omega, y_n(\omega))\|}{1 + M\|S(\omega, x(\omega)) - T(\omega, x(\omega))\|}\right] \\ &\le (1-\alpha_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \alpha_n a \|S(\omega, y_n(\omega)) - p(\omega)\|, \end{aligned} \tag{2.2}$$

for estimate $\|S(\omega, y_n(\omega)) - p(\omega)\|$ in (2.2), we can write

$$\begin{aligned} \|S(\omega, y_n(\omega)) - p(\omega)\| &= \|(1-\beta_n)S(\omega, x_n(\omega)) + \beta_n T(\omega, x_n(\omega)) - p(\omega)\| \\ &= \|(1-\beta_n)(S(\omega, x_n(\omega)) - p(\omega)) + \beta_n(T(\omega, x_n(\omega)) - p(\omega))\| \\ &\le (1-\beta_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \beta_n \|T(\omega, x_n(\omega)) - p(\omega)\|, \end{aligned}$$

again, using (2.1) and (1.7), we have

$$\begin{aligned} \|S(\omega, y_n(\omega)) - p(\omega)\| &\le (1-\beta_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \beta_n a \|S(\omega, x_n(\omega)) - p(\omega)\| \\ &\le [1 - \beta_n(1-a)]\|S(\omega, x_n(\omega)) - p(\omega)\|. \end{aligned} \tag{2.3}$$

Applying (2.3) in (2.2), we get

$$\|S(\omega, x_{n+1}(\omega)) - p(\omega)\| \le [1 - (1-a)\alpha_n - (1-a)a\alpha_n\beta_n]\|S(\omega, x_n(\omega)) - p(\omega)\|, \tag{2.4}$$

since $\beta_n \to 0$ as $n \to \infty$, $a \in [0,1)$ and $0 < \alpha_n < 1$, so $1 - (1-a)\alpha_n = h$, where $0 < h < 1$, hence (2.4) being

$$\begin{aligned} \|S(\omega, x_{n+1}(\omega)) - p(\omega)\| &\le [1 - (1-a)\alpha_n]\|S(\omega, x_n(\omega)) - p(\omega)\| \le h\|S(\omega, x_n(\omega)) - p(\omega)\| \\ &\le h^n \|S(\omega, x_\circ(\omega)) - p(\omega)\| \to 0 \text{ as } n \to \infty. \end{aligned}$$

Therefore, we obtain from above inequality $\|S(\omega, x_{n+1}(\omega)) - p(\omega)\| \to 0$, i.e. $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ converges strongly to $p(\omega)$.From (1.5) and (1.7), for $\omega \in \Omega$, we obtain that

$$\begin{aligned}
\|p(\omega) - T(\omega, p(\omega))\| &\leq \|p(\omega) - S(\omega, x_{n+1}(\omega))\| + \|S(\omega, x_{n+1}(\omega)) - T(\omega, p(\omega))\| \\
&\leq \|p(\omega) - S(\omega, x_{n+1}(\omega))\| + \|(1-\alpha_n)S(\omega, x_n(\omega)) + \alpha_n T(\omega, y_n(\omega)) - T(\omega, p(\omega))\| \\
&\leq \|p(\omega) - S(\omega, x_{n+1}(\omega))\| + (1-\alpha_n)\|S(\omega, x_n(\omega)) - T(\omega, p(\omega))\| \\
&\quad + \alpha_n \|T(\omega, y_n(\omega)) - T(\omega, p(\omega))\| \\
&\leq \|p(\omega) - S(\omega, x_{n+1}(\omega))\| + (1-\alpha_n)\|S(\omega, x_n(\omega)) - T(\omega, p(\omega))\| \\
&\quad + \alpha_n \left(\frac{\varphi(\|S(\omega, p(\omega)) - T(\omega, p(\omega))\|) + a\|S(\omega, p(\omega)) - S(\omega, y_n(\omega))\|}{1 + M\|S(\omega, p(\omega)) - T(\omega, p(\omega))\|} \right),
\end{aligned}$$

since S and T have a random coincidence point and continuous mappings i.e. $S(\omega, p(\omega)) = T(\omega, p(\omega))$, therefore

$$\begin{aligned}
\|p(\omega) - T(\omega, p(\omega))\| &\leq \|p(\omega) - S(\omega, x_{n+1}(\omega))\| + (1-\alpha_n)\|S(\omega, x_n(\omega)) - T(\omega, p(\omega))\| \\
&\quad + \alpha_n a[\|S(\omega, p(\omega)) - S(\omega, y_n(\omega))\|] \\
&\leq \|p(\omega) - S(\omega, x_{n+1}(\omega))\| + (1-\alpha_n)\|S(\omega, x_n(\omega)) - T(\omega, p(\omega))\| \\
&\quad + \alpha_n a \left[\begin{array}{c} (1-\beta_n)\|S(\omega, x_n(\omega)) - S(\omega, p(\omega))\| \\ + \beta_n \|T(\omega, x_n(\omega)) - S(\omega, p(\omega))\| \end{array} \right],
\end{aligned}$$

taking limit in the above inequality and using $\beta_n \to 0$ as $n \to \infty$ and $p(\omega) = S(\omega, x_{n+1}(\omega))$, one can write

$$\begin{aligned}
\|p(\omega) - T(\omega, p(\omega))\| &\leq (1-\alpha_n)\|p(\omega) - T(\omega, p(\omega))\| + \alpha_n a\|p(\omega) - S(\omega, p(\omega))\| \\
&\leq (1-\alpha_n)\|p(\omega) - T(\omega, p(\omega))\| + \alpha_n a\|p(\omega) - T(\omega, p(\omega))\|
\end{aligned}$$

So,

$$\alpha_n(1-a)\|p(\omega) - T(\omega, p(\omega))\| \leq 0,$$

since $\alpha_n(1-a) > 0$, we get $\|p(\omega) - T(\omega, p(\omega))\| \leq 0$. Hence, $p(\omega) = T(\omega, p(\omega))$ for all $\omega \in \Omega$.Again since $S(\omega, p(\omega)) = T(\omega, p(\omega))$, so $p(\omega) = T(\omega, p(\omega)) = S(\omega, p(\omega))$. Thus for all $\omega \in \Omega$, $p(\omega)$ is common random fixed point of S and T.

For a uniqueness. Let $q(\omega)$ be another common random fixed point of S and T such that $q(\omega) \neq p(\omega)$, then by using (1.7), we have

$$\begin{aligned}
\|p(\omega) - q(\omega)\| &= \|T(\omega, p(\omega)) - T(\omega, q(\omega))\| \\
&\leq \frac{\varphi(\|S(\omega, p(\omega)) - T(\omega, p(\omega))\|) + a\|S(\omega, p(\omega)) - S(\omega, q(\omega))\|}{1 + M\|S(\omega, p(\omega)) - T(\omega, p(\omega))\|} \\
&\leq a\|p(\omega) - q(\omega)\|,
\end{aligned}$$

since $0 \leq a < 1$, therefore $p(\omega) = q(\omega)$. □

Theorem 2.2. *Let X be a separable Banach space, C be a nonempty closed convex subset of X. Let $T, S : \Omega \times C \to X$ be two non self continuous random operators defined on C with $T(\omega, C) \subseteq S(\omega, C)$ satisfying (1.7). Assume that random operators S and T have a random coincidence point. If the sequence $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ defined by (1.6) converges, then converges to a unique common random fixed point of S and T.*

Proof. By a similar manner in proving Theorem 2.1, we shall prove that the random Jungck-Noor iteration $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ strongly convergence to $p(\omega)$, we have from (1.6) for $\omega \in \Omega$,

$$\begin{aligned}
\|S(\omega, x_{n+1}(\omega)) - p(\omega)\| &= \|(1-\alpha_n)S(\omega, x_n(\omega)) + \alpha_n T(\omega, z_n(\omega)) - p(\omega)\| \\
&= \|(1-\alpha_n)(S(\omega, x_n(\omega)) - p(\omega)) + \alpha_n(T(\omega, z_n(\omega)) - p(\omega))\| \\
&\leq (1-\alpha_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \alpha_n\|T(\omega, z_n(\omega)) - p(\omega)\|,
\end{aligned}$$

Applying (2.1) and (1.7), we obtain that

$$
\begin{aligned}
\|S(\omega, x_{n+1}(\omega)) - p(\omega)\| &\leq (1-\alpha_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \alpha_n \|T(\omega, z_n(\omega)) - T(\omega, x(\omega))\| \\
&\leq (1-\alpha_n)\|S(\omega, x_n(\omega)) - p(\omega)\| \\
&+ \alpha_n \left(\frac{\varphi(\|S(\omega, x(\omega)) - T(\omega, x(\omega))\| + a\|S(\omega, x(\omega)) - S(\omega, z_n(\omega))\|}{1 + M\|S(\omega, x(\omega)) - T(\omega, x(\omega))\|} \right) \\
&= (1-\alpha_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \alpha_n a \|S(\omega, z_n(\omega)) - p(\omega)\|, \qquad (2.5)
\end{aligned}
$$

for estimate $\|S(\omega, z_n(\omega)) - p(\omega)\|$ in (2.5), we get

$$
\begin{aligned}
\|S(\omega, z_n(\omega)) - p(\omega)\| &= \|(1-\beta_n)S(\omega, x_n(\omega)) + \beta_n T(\omega, y_n(\omega)) - p(\omega)\| \\
&= \|(1-\beta_n)(S(\omega, x_n(\omega)) - p(\omega)) + \beta_n (T(\omega, y_n(\omega)) - p(\omega))\| \\
&\leq (1-\beta_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \beta_n \|T(\omega, y_n(\omega)) - p(\omega)\|,
\end{aligned}
$$

again, using (2.1) and (1.7), we deduce that

$$
\|S(\omega, z_n(\omega)) - p(\omega)\| \leq (1-\beta_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \beta_n a \|S(\omega, y_n(\omega)) - p(\omega)\|, \qquad (2.6)
$$

introducing (2.6) into (2.5), yields

$$
\begin{aligned}
\|S(\omega, x_{n+1}(\omega)) - p(\omega)\| &\leq (1-\alpha_n)\|S(\omega, x_n(\omega)) - p(\omega)\| \\
&+ \alpha_n a \left[(1-\beta_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \beta_n a \|S(\omega, y_n(\omega)) - p(\omega)\|\right] \\
&\leq [1 - \alpha_n(1-a) - \alpha_n \beta_n a]\|S(\omega, x_n(\omega)) - p(\omega)\| \\
&+ \alpha_n \beta_n a^2 \|S(\omega, y_n(\omega)) - p(\omega)\|, \qquad (2.7)
\end{aligned}
$$

for estimate $\|S(\omega, y_n(\omega)) - p(\omega)\|$ in (2.7), using (2.1) and (1.7), we can write

$$
\begin{aligned}
\|S(\omega, y_n(\omega)) - p(\omega)\| &= \|(1-\gamma_n)S(\omega, x_n(\omega)) + \gamma_n T(\omega, x_n(\omega)) - p(\omega)\| \\
&= \|(1-\gamma_n)(S(\omega, x_n(\omega)) - p(\omega)) + \gamma_n (T(\omega, x_n(\omega)) - p(\omega))\| \\
&\leq (1-\gamma_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \gamma_n \|T(\omega, x_n(\omega)) - p(\omega)\| \\
&\leq (1-\gamma_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \gamma_n \|T(\omega, x(\omega)) - T(\omega, x_n(\omega))\| \\
&\leq (1-\gamma_n)\|S(\omega, x_n(\omega)) - p(\omega)\| + \gamma_n a \|S(\omega, x_n(\omega)) - p(\omega)\| \\
&\leq [1 - \gamma_n(1-a)]\|S(\omega, x_n(\omega)) - p(\omega)\|. \qquad (2.8)
\end{aligned}
$$

Applying (2.8) in (2.7), it follows that

$$
\begin{aligned}
\|S(\omega, x_{n+1}(\omega)) - p(\omega)\| &\leq [1 - (1-a)\alpha_n - (1-a)\alpha_n\beta_n a - (1-a)\alpha_n\beta_n\gamma a^2]\|S(\omega, x_n(\omega)) - p(\omega)\| \\
&\leq [1 - (1-a)\alpha_n]\|S(\omega, x_n(\omega)) - p(\omega)\| \\
&\leq h\|S(\omega, x_n(\omega)) - p(\omega)\| \leq h^n \|S(\omega, x_\circ(\omega)) - p(\omega)\|.
\end{aligned}
$$

Since $0 \leq h < 1$, taking the limit as $n \to \infty$ in the above inequality, then we have $h^n \|S(\omega, x_\circ(\omega)) - p(\omega)\| \to 0$ as $n \to \infty$. Which mean that $\lim_{n\to\infty} \|S(\omega, x_{n+1}(\omega)) - p(\omega)\| = 0$. Therefore random Jungck-Noor iteration $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ converges strongly to $p(\omega)$. The proof of a unique common random fixed point is the same as Theorem 2.1. □

Finally, we will give an example to support our theorems.

Example 2.3. Let (Ω, Σ) denotes a measurable space, $C = \Omega = \{1, 2, 3, 4\} \subset X = \mathbb{R}$ with the usual metric d and $\sum$ be the sigma algebra of Lebesgue's measurable subset of Ω. For all $\omega \in \Omega$, define $T, S : \Omega \times C \to C$ by

$$
T(\omega, x) = \begin{cases} 2 & x = 1, \\ 4 & \text{otherwise}, \end{cases} \qquad S(\omega, x) = \begin{cases} 3 & x = 1, \\ 4 & \text{otherwise}, \end{cases}
$$

It's clearly $T(\omega, C) \subseteq S(\omega, C)$ and the contractive condition (1.7) is satisfied if we take $x = y = 1$ or $x = y \in C - \{1\}$. Assume that $\alpha_n = \sqrt{1 - \frac{1}{n}}$, $\beta_n = \frac{e^n}{e^n+1}$ and $\gamma_n = \frac{n-1}{n+5}$, $n \geq 1$ then the sequence $\{S(\omega, x_n(\omega))\}$ defined by (1.5) or (1.6) converges to a unique random fixed point 4 of S and T.

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