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Intuitionistic fuzzy Zweier I-convergent sequence spaces

Vakeel A. Khan*, Khalid Ebadullah, Rami Kamel Ahmad Rababah

Department of Mathematics, Aligarh Muslim University, Aligarh-202002, India.

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Abstract

In this article we introduce the intuitionistic fuzzy Zweier I-convergent sequence spaces $\mathcal{Z}^I_{(\mu,\nu)}$ and $\mathcal{Z}^I_{0(\mu,\nu)}$ and study the fuzzy topology on the said spaces.

Keywords: Ideal, filter, I-convergence, intuitionistic fuzzy normed space.
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1. Introduction

The fuzzy theory has emerged as the most active area of research in many branches of science and engineering. Among various developments of the theory of fuzzy sets [16] a progressive development has been made to find the fuzzy analogues of the classical set theory. In fact the fuzzy theory has become an area of active research for the last 50 years. It has a wide range of applications in the field of science and engineering, e.g. application of fuzzy topology in quantum particle physics that arises in string and $e^{(\infty)}$-theory of El-Naschie [3]-[7], chaos control, computer programming, nonlinear dynamical system and population dynamics etc.

In many branches of science and engineering we often come across with different type of sequences and certainly there are situations of inexactness where the idea of ordinary convergence does not work. So to deal with such situations we have to introduce new measures and tools which is suitable to the said situation. That is we are interested to put forward our studies in fuzzy like situations. Here we recall some notations and basic definitions which we need throughout the work.

Definition 1.1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be continuous t-norm if it satisfies the following conditions:

*Corresponding author
Email addresses: vakhanmaths@gmail.com (Vakeel A. Khan), khalidebadullah@gmail.com (Khalid Ebadullah)

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(a) * is associative and commutative;

(b) * is continuous;

(c) $a * 1 = a$ for all $a \in [0, 1]$;

(d) $a * c \leq b * d$ whenever $a \leq b$ and $c \leq d$ for each $a, b, c, d \in [0, 1]$.

For example, $a * b = a.b$ is a continuous t-norm.

Definition 1.2. A binary operation $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ is said to be continuous t-conorm if it satisfies the following conditions:

(a) $\diamond$ is associative and commutative;

(b) $\diamond$ is continuous;

(c) $a \diamond 0 = a$ for all $a \in [0, 1]$;

(d) $a \diamond c \leq b \diamond d$ whenever $a \leq b$ and $c \leq d$ for each $a, b, c, d \in [0, 1]$.

For example, $a \diamond b = min\{a + b, 1\}$ is a continuous t-conorm.

Definition 1.3. Let * be a continuous t-norm and $\diamond$ be a continuous t-conorm and X be a linear space over the field($\mathbb{R}$ or $\mathbb{C}$). If μ and ν are fuzzy sets on $X \times (0, \infty)$ satisfying the following conditions, the five- tuple $(X, \mu, \nu, *, \diamond)$ is said to be an intuitionistic fuzzy normed space (IFNS) and (μ,ν) is called an intuitionistic fuzzy norm. For every $x, y \in X$ and $s, t > 0$,

(a) $\mu(x, t) + \nu(x, t) \leq 1$,

(b) $\mu(x, t) > 0$,

(c) $\mu(x, t) = 1$ iff $x = 0$,

(d) $\mu(ax, t) = \mu(x, \frac{t}{|a|})$ for each $a \neq 0$,

(e) $\mu(x, t) * \mu(y, s) \leq \mu(x + y, t + s)$,

(f) $\mu(x, .) : (0, \infty) \to [0, 1]$ is continuous,

(g) $\lim_{t\to\infty} \mu(x, t) = 1$ and $\lim_{t\to 0} \mu(x, t) = 0$,

(h) $\nu(x, t) < 1$,

(i) $\nu(x, t) = 0$ iff $x = 0$,

(j) $\nu(ax, t) = \nu(x, \frac{t}{|a|})$ for each $a \neq 0$,

(k) $\nu(x, t) \diamond \nu(y, s) \geq \nu(x + y, t + s)$,

(l) $\nu(x, .) : (0, \infty) \to [0, 1]$ is continuous,

(m) $\lim_{t\to\infty} \nu(x, t) = 1$ and $\lim_{t\to 0} \nu(x, t) = 0$,

(n) $a * a = a, a \diamond a = a$ for all $a \in [0, 1]$.

Definition 1.4. Let $(X, \mu, \nu, *, \diamond)$ be IFNS and (x_n) be a sequence in X. Sequence (x_n) is said to be convergent to $L \in X$ with respect to the intuitionistic fuzzy norm (μ,ν) if for every $\epsilon > 0$ and $t > 0$, there exists a positive integer n_0 such that $\mu(x_n - L, t) > 1 - \epsilon$ and $\nu(x_n - L, t) < \epsilon$ whenever $n > n_0$. In this case we write $(\mu,\nu) - \lim x_n = L$ as $n \to \infty$.

Definition 1.5. If X be a non- empty set, then a family of set $I \subset P(X)(P(X)$ denoting the power set of $X)$ is called an ideal in X if and only if

(a) $\phi \in I$;

(b) For each $A, B \in I$, we have $A \cup B \in I$;

(c) For each $A \in I$ and $B \subset A$ we have $B \in I$.

Definition 1.6. If X be a non-empty set. A non-empty family of sets $F \subset P(X)(P(X)$ denoting the power set of $X)$ is called a filter on X if and only if

(a) $\phi \notin F$;

(b) For each $A, B \in F$, we have $A \cap B \in F$;

(c) For each $A \in F$ and $A \subset B$ we have $B \in F$.

Definition 1.7. Let $I \subset P(N)$ be a non trivial ideal and $(X, \mu, \nu, *, \diamond)$ be an IFNS. A sequence $x = (x_n)$ of elements in X is said to be I-convergent to $L \in X$ with respect to the intuitionistic fuzzy norm (μ,ν) if for every $\epsilon > 0$ and $t > 0$, the set

$$\{n \in N : \mu(x_n - L, t) \geq 1 - \epsilon \text{ or } \nu(x_n - L, t) \leq \epsilon\} \in I.$$

In this case L is called the I-limit of the sequence (x_n) with respect to the intuitionistic fuzzy norm (μ, ν) and we write $I_{(\mu,\nu)}$- $\lim x_n = L$.

Definition 1.8. Let $(X, \mu, \nu, *, \diamond)$ be an IFNS. Let $r \in (0, 1)$, $t > 0$ and $x \in X$. The set

$$B_x(r, t) = \{k \in N : \mu(x_k - L, t) \leq 1 - r \text{ or } \nu(x_k - L, t) \geq r\} \in I$$

is called an open ball with center x and radius r with respect to t. (c.f. [9],[12],[13],[16])

The approach of constructing new sequence spaces by means of the matrix domain of a particular limitation method have been recently employed by Altay, Başar and Mursaleen [1], Başar and Altay [2], Malkowsky [10], Ng and Lee [11], and Wang [15]. Şengönül [14] defined the sequence $y = (y_i)$ which is frequently used as the Z^p transform of the sequence $x = (x_i)$ i.e,

$$y_i = px_i + (1 - p)x_{i-1},$$

where $x_{-1} = 0, p \neq 1$, $1 < p < \infty$ and Z^p denotes the matrix $Z^p = (z_{ik})$ defined by

$$z_{ik} = \begin{cases} p, & (i = k), \\ 1 - p, & (i - 1 = k); (i, k \in \mathbb{N}), \\ 0, & \text{otherwise.} \end{cases}$$

Analogous to Başar and Altay [2], Şengönül [14] introduced the Zweier sequence spaces $\mathcal{Z}$ and $\mathcal{Z}_0$ as follows:

$$\mathcal{Z} = \{x = (x_k) \in \omega : Z^p x \in c\};$$

$$\mathcal{Z}_0 = \{x = (x_k) \in \omega : Z^p x \in c_0\}.$$

Recently Khan, Ebadullah and Yasmeen [8] introduced the following classes of sequences,

$$\mathcal{Z}^I = \{(x_k) \in \omega : \text{there exists } L \in \mathbb{C} \text{ such that for a given } \varepsilon > 0, \{k \in \mathbb{N} : |x'_k - L| \geq \varepsilon\} \in I\};$$
$$\mathcal{Z}^I_0 = \{(x_k) \in \omega : \text{for a given } \varepsilon > 0, \{k \in \mathbb{N} : |x'_k| \geq \varepsilon\} \in I\},$$

where $(x'_k) = (Z^p x)$.

In this article we introduce the intuitionistic fuzzy Zweier I-convergent sequence spaces as follows:

$$\mathcal{Z}^I_{(\mu,\nu)} = \{\{k \in N : \mu(x'_k - L, t) \leq 1 - \epsilon \text{ or } \nu(x'_k - L, t) \geq \epsilon\} \in I\};$$
$$\mathcal{Z}^I_{0(\mu,\nu)} = \{\{k \in N : \mu(x'_k, t) \leq 1 - \epsilon \text{ or } \nu(x'_k, t) \geq \epsilon\} \in I\}.$$

2. Main Results

Theorem 2.1. *$\mathcal{Z}^I_{(\mu,\nu)}$ and $\mathcal{Z}^I_{0(\mu,\nu)}$ are linear spaces.*

Proof. We prove the result for $\mathcal{Z}^I_{(\mu,\nu)}$. Similarly the result can be proved for $\mathcal{Z}^I_{0(\mu,\nu)}$. Let $(x'_k), (y'_k) \in \mathcal{Z}^I_{(\mu,\nu)}$ and let α, β be scalars. Then for a given $\epsilon > 0$. For $\alpha = 0, \beta = 0$ the proof is trivial, we have

$$\begin{aligned}
A_1 &= \{k \in N : \mu(x'_k - L_1, \frac{t}{2|\alpha|}) \leq 1 - \epsilon \text{ or } \nu(x'_k - L_1, \frac{t}{2|\alpha|}) \geq \epsilon\} \in I,\\
A_2 &= \{k \in N : \mu(y'_k - L_2, \frac{t}{2|\beta|}) \leq 1 - \epsilon \text{ or } \nu(y'_k - L_2, \frac{t}{2|\beta|}) \geq \epsilon\} \in I,\\
A_1^c &= \{k \in N : \mu(x'_k - L_1, \frac{t}{2|\alpha|}) > 1 - \epsilon \text{ or } \nu(x'_k - L_1, \frac{t}{2|\alpha|}) < \epsilon\} \in F(I),\\
A_2^c &= \{k \in N : \mu(y'_k - L_2, \frac{t}{2|\beta|}) > 1 - \epsilon \text{ or } \nu(y'_k - L_2, \frac{t}{2|\beta|}) < \epsilon\} \in F(I).
\end{aligned}$$

Define the set $A_3 = A_1 \cup A_2$, so that $A_3 \in I$. It follows that A_3^c is a non-empty set in $F(I)$. We shall show that for each $(x'_k), (y'_k) \in \mathcal{Z}^I_{(\mu,\nu)}$.

$$A_3^c \subset \{k \in N : \mu((\alpha x'_k + \beta y'_k) - (\alpha L_1 + \beta L_2), t) > 1 - \epsilon \text{ or } \nu((\alpha x'_k + \beta y'_k) - (\alpha L_1 + \beta L_2), t) < \epsilon\}.$$

Let $m \in A_3^c$. In this case

$$\mu(x'_m - L_1, \frac{t}{2|\alpha|}) > 1 - \epsilon \text{ or } \nu(x'_m - L_1, \frac{t}{2|\alpha|}) < \epsilon$$

and

$$\mu(y'_m - L_2, \frac{t}{2|\beta|}) > 1 - \epsilon \text{ or } \nu(y'_m - L_2, \frac{t}{2|\beta|}) < \epsilon.$$

We have

$$\begin{aligned}
\mu((\alpha x'_m + \beta y'_m) - (\alpha L_1 + \beta L_2), t) &\geq \mu(\alpha x'_m - \alpha L_1, \frac{t}{2}) * \mu(\beta y'_m - \beta L_2, \frac{t}{2})\\
&= \mu(x'_m - L_1, \frac{t}{2|\alpha|}) * \mu(y'_m - L_2, \frac{t}{2|\beta|})\\
&> (1 - \epsilon) * (1 - \epsilon)\\
&= (1 - \epsilon)
\end{aligned}$$

and

$$\begin{aligned}
\nu((\alpha x'_m + \beta y'_m) - (\alpha L_1 + \beta L_2), t) &\leq \nu(\alpha x'_m - \alpha L_1, \frac{t}{2}) \diamond \nu(\beta y'_m - \beta L_2, \frac{t}{2})\\
&= \nu(x'_m - L_1, \frac{t}{2|\alpha|}) \diamond \nu(y'_m - L_2, \frac{t}{2|\beta|})\\
&< \epsilon \diamond \epsilon\\
&= \epsilon.
\end{aligned}$$

This implies that

$$A_3^c \subset \{k \in N : \mu((\alpha x'_k + \beta y'_k) - (\alpha L_1 + \beta L_2), t) > 1 - \epsilon \text{ or } \nu((\alpha x'_k + \beta y'_k) - (\alpha L_1 + \beta L_2), t) < \epsilon\}.$$

Hence $\mathcal{Z}^I_{(\mu,\nu)}$ is a linear space. □

Theorem 2.2. *Every open ball $B_{x'}(r, t)$ is an open set in $\mathcal{Z}^I_{(\mu,\nu)}$.*

Proof. Let $B_{x'}(r,t)$ be an open ball with center x' and radius r with respect to t. That is

$$B_{x'}(r,t) = \{k \in N : \mu(x_{k'} - L, t) \leq 1 - r \text{ or } \nu(x_{k'} - L, t) \geq r\} \in I.$$

Let $y' \in B^c_{x'}(r,t)$. Then $\mu(x'-y',t) > 1-r$ and $\nu(x'-y',t) < r$. Since $\mu(x'-y',t) > 1-r$, there exists $t_0 \in (0,t)$ such that $\mu(x'-y',t_0) > 1-r$ and $\nu(x'-y',t_0) < r$. Putting $r_0 = \mu(x'-y',t_0)$, we have $r_0 > 1-r$, there exists $s \in (0,1)$ such that $r_0 > 1-s > 1-r$. For $r_0 > 1-s$ we have $r_1, r_2 \in (0,1)$ such that $r_0 * r_1 > 1-s$ and $(1-r_0) \diamond (1-r_2) \leq s$. Putting $r_3 = max\{r_1, r_2\}$, consider the ball $B^c_{y'}(1-r_3, t-t_0)$. We prove that $B^c_{y'}(1-r_3, t-t_0) \subset B^c_{x'}(r,t)$. Let $z' \in B^c_{y'}(1-r_3, t-t_0)$, $\mu(y'-z', t-t_0) > r_3$ and $\nu(y'-z', t-t_0) < r_3$. Therefore

$$\mu(x'-z',t) \geq \mu(x'-y',t_0) * \mu(y'-z',t-t_0) \geq (r_0 * r_3) \geq (r_0 * r_1) \geq (1-s) > (1-r)$$

and

$$\nu(x'-z',t) \leq \nu(x'-y',t_0) \diamond \nu(y'-z',t-t_0) \leq (1-r_0) \diamond (1-r_3) \leq (1-r_0) \diamond (1-r_2) \leq s < r.$$

Thus $z' \in B^c_{x'}(r,t)$ and hence $B^c_{y'}(1-r_3, t-t_0) \subset B^c_{x'}(r,t)$. □

Remark 2.3. $\mathcal{Z}^I_{(\mu,\nu)}$ is an IFNS. Define

$$\tau_{(\mu,\nu)} = \{A \subset \mathcal{Z}^I_{(\mu,\nu)} : \text{for each } x \in A \text{ there exists } \ t > 0 \text{ and } r \in (0,1) \text{ such that } B_{x'}(y',t) \subset A\}.$$

Then $\tau_{(\mu,\nu)}$ is a topology on $\mathcal{Z}^I_{(\mu,\nu)}$.

Theorem 2.4. *The topology $\tau_{(\mu,\nu)}$ on $\mathcal{Z}^I_{0(\mu,\nu)}$ is first countable.*

Proof. $\{B_{x'}(\frac{1}{n}, \frac{1}{n})$: n=1, 2, 3 ....$\}$ is a local base at x', the topology $\tau_{(\mu,\nu)}$ on $\mathcal{Z}^I_{0(\mu,\nu)}$ is first countable. □

Theorem 2.5. *$\mathcal{Z}^I_{(\mu,\nu)}$ and $\mathcal{Z}^I_{0(\mu,\nu)}$ are Hausdorff spaces.*

Proof. We prove the result for $\mathcal{Z}^I_{(\mu,\nu)}$. Similarly the proof follows for $\mathcal{Z}^I_{0(\mu,\nu)}$. Let $x', y' \in \mathcal{Z}^I_{(\mu,\nu)}$ such that $x' \neq y'$. Then $0 < \mu(x'-y',t) < 1$ and $0 < \nu(x'-y',t) < 1$. Putting $r_1 = \mu(x'-y',t)$ and $r_2 = \nu(x'-y',t)$ and $r = max\{r_1, 1-r_2\}$. For each $r_0 \in (r,1)$, there exists r_3 and r_4 such that $r_3 * r_4 \geq r_0$ and $(1-r_3) \diamond (1-r_4) \leq (1-r_0)$. Putting $r_5 = max\{r_3, 1-r_4\}$ and consider the open balls $B_{x'}(1-r_5, \frac{t}{2})$ and $B_{y'}(1-r_5, \frac{t}{2})$. Then clearly $B^c_{x'}(1-r_5, \frac{t}{2}) \cap B^c_{y'}(1-r_5, \frac{t}{2}) = \phi$. For if there exists $z' \in B^c_{x'}(1-r_5, \frac{t}{2}) \cap B^c_{y'}(1-r_5, \frac{t}{2})$ then

$$r_1 = \mu(x'-y',t) \geq \mu(x'-z', \frac{t}{2}) * \mu(z'-y', \frac{t}{2}) \geq r_5 * r_5 \geq r_3 * r_3 \geq r_0 > r_1$$

and

$$r_2 = \nu(x'-y',t) \leq \nu(x'-z', \frac{t}{2}) \diamond \nu(z'-y', \frac{t}{2}) \leq (1-r_5) \diamond (1-r_5) \leq (1-r_4) \diamond (1-r_4) \leq (1-r_0) < r_2,$$

which is a contradiction. Hence $\mathcal{Z}^I_{(\mu,\nu)}$ is Hausdorff. □

Theorem 2.6. *$\mathcal{Z}^I_{(\mu,\nu)}$ is an IFNS. $\tau_{(\mu,\nu)}$ is a topology on $\mathcal{Z}^I_{(\mu,\nu)}$.Then a sequence $(x'_k) \in \mathcal{Z}^I_{(\mu,\nu)}$, $x'_k \to x'$ if and only if $\mu(x'_k - x', t) \to 1$ and $\nu(x'_k - x', t) \to 0$ as $k \to \infty$.*

Proof. Fix $t_0 > 0$. Suppose $x'_k \to x'$. Then for $r \in (0,1)$, there exists $n_0 \in N$ such that $x'_k \in B_{x'}(r,t)$ for all $k \geq n_0$.

$$B_{x'}(r,t) = \{k \in N : \mu(x'_k - x', t) \leq 1 - r \text{ or } \nu(x'_k - x', t) \geq r\} \in I,$$

such that $B^c_{x'}(r,t) \in F(I)$. Then $1-\mu(x'_k - x', t) < r$ and $\nu(x'_k - x', t) < r$. Hence $\mu(x'_k - x', t) \to 1$ and $\nu(x'_k - x', t) \to 0$ as $k \to \infty$.

Conversely, if for each $t > 0$ $\mu(x'_k - x', t) \to 1$ and $\nu(x'_k - x', t) \to 0$ as $k \to \infty$, then for $r \in (0,1)$, there exists $n_0 \in N$ such that $1-\mu(x'_k - x', t) < r$ and $\nu(x'_k - x', t) < r$, for all $k \geq n_0$. It follows that $\mu(x'_k - x', t) > 1-r$ and $\nu(x'_k - x', t) < r$ for all $k \geq n_0$. Thus $x'_k \in B^c_{x'}(r,t)$ for all $k \geq n_0$ and hence $x'_k \to x'$. □

Theorem 2.7. *A sequence $x = (x'_k) \in \mathcal{Z}^I_{(\mu,\nu)}$ is I-converges if and only if for every $\epsilon > 0$ and $t > 0$ there exists a number $N = N(x, \epsilon, t)$ such that*

$$\{k \in N : \mu(x'_k - L, \frac{t}{2}) > 1-\epsilon \text{ or } \nu(x'_k - L, \frac{t}{2}) < \epsilon\} \in F(I).$$

Proof. Suppose that $I_{(\mu,\nu)} - x = L$ and let $\epsilon > 0$ and $t > 0$. For a given $\epsilon > 0$, choose $s > 0$ such that $(1-\epsilon) * (1-\epsilon) > 1-s$ and $\epsilon \diamond \epsilon < s$. Then for each $x \in \mathcal{Z}^I_{(\mu,\nu)}$,

$$A_x(\epsilon,t) = \{k \in N : \mu(x'_k - L, \frac{t}{2}) \leq 1-\epsilon \text{ or } \nu(x'_k - L, \frac{t}{2}) \geq \epsilon\} \in I,$$

which implies that

$$A^c_x(\epsilon,t) = \{k \in N : \mu(x'_k - L, \frac{t}{2}) > 1-\epsilon \text{ or } \nu(x'_k - L, \frac{t}{2}) < \epsilon\} \in F(I).$$

Conversly let us choose $N \in A^c_x(\epsilon,t)$. Then

$$\mu(x'_N - L, \frac{t}{2}) > 1-\epsilon \text{ or } \nu(x'_N - L, \frac{t}{2}) < \epsilon.$$

Now we want to show that there exists a number $N = N(x, \epsilon, t)$ such that

$$\{k \in N : \mu(x'_k - x'_N, t) \leq 1-s \text{ or } \nu(x'_k - x'_N, t) \geq s\} \in I.$$

For this, define for each $x \in \mathcal{Z}^I_{(\mu,\nu)}$

$$B_x(\epsilon,t) = \{k \in N : \mu(x'_k - x'_N, t) \leq 1-s \text{ or } \nu(x'_k - x'_N, t) \geq s\} \in I.$$

Now we show that $B_x(\epsilon,t) \subset A_x(\epsilon,t)$. Suppose that $B_x(\epsilon,t) \not\subset A_x(\epsilon,t)$. Then there exists $n \in B_x(\epsilon,t)$ and $n \notin A_x(\epsilon,t)$. Therefore we have

$$\mu(x'_n - x'_N, t) \leq 1-s \text{ and } \mu(x'_n - L, \frac{t}{2}) > 1-\epsilon.$$

In particular $\mu(x'_N - L, \frac{t}{2}) > 1-\epsilon$. Therefore we have

$$1-s \geq \mu(x'_n - x'_N, t) \geq \mu(x'_n - L, \frac{t}{2}) * \mu(x'_N - L, \frac{t}{2}) \geq (1-\epsilon) * (1-\epsilon) > 1-s,$$

which is not possible. On the other hand

$$\nu(x'_n - x'_N, t) \geq s \text{ and } \nu(x'_n - L, \frac{t}{2}) < \epsilon.$$

In particular $\nu(x'_N - L, \frac{t}{2}) < \epsilon$. Therefore we have

$$s \leq \nu(x'_n - x'_N, t) \leq \nu(x'_n - L, \frac{t}{2}) \diamond \nu(x'_N - L, \frac{t}{2}) \leq \epsilon \diamond \epsilon < s,$$

which is not possible. Hence $B_x(\epsilon,t) \subset A_x(\epsilon,t)$. $A_x(\epsilon,t) \in I$ implies $B_x(\epsilon,t) \in I$. □

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