Available online at www.vonneumann-publishing.com/fatma
Func. Anal.-TMA 2 (2016), 56–68
Research Article

Functional Analysis: Theory, Methods & Applications
Journal Homepage: http://vonneumann-publishing.com/fatma

Solvability of iterative systems of nonlinear m-point boundary value problems on time scales

Kapula Rajendra Prasad[a], Namburi Sreedhar[b,*]

[a]Department of Applied Mathematics, Andhra University, Visakhapatnam, 530 003, India.
[b]Department of Mathematics, GITAM University, Visakhapatnam, 530 045, India.

Communicated by R. Saadati

Abstract

In this paper, we consider the iterative system of nonlinear m-point boundary value problems on time scales,

$$y_i^{\Delta\Delta}(t) + \lambda_i p_i(t) f_i(y_{i+1}(t)) = 0, \quad 1 \le i \le n, \quad t \in [t_1, \sigma(t_m)]_{\mathbb{T}},$$
$$y_{n+1}(t) = y_1(t), \quad t \in [t_1, \sigma(t_m)]_{\mathbb{T}},$$

$$\alpha_i y_i(t_1) - \beta_i y_i^{\Delta}(t_1) = 0,$$
$$\gamma_i y_i(\sigma(t_m)) + \delta_i y_i^{\Delta}(\sigma(t_m)) = \sum_{k=2}^{m-1} y_i^{\Delta}(t_k), \quad 1 \le i \le n.$$

We express the solution of the above boundary value problem in to an equivalent integral equation involving Green functions and obtain the bounds for these Green functions. By applying Guo–Krasnosel'skiı̆ fixed point theorem, we determine the eigenvalue intervals of λ_i, $1 \le i \le n$, for the existence of at least of one positive solution of the boundary value problem. As an application, we give an example to demonstrate our results.

Keywords: Green's function, iterative system, boundary value problem, time scale, m-point, eigenvalue interval, positive solution, cone.
2010 MSC: 39A10, 34B18, 34A40.

*Corresponding author
Email addresses: rajendra92@rediffmail.com (Kapula Rajendra Prasad), sreedharnamburi@rediffmail.com (Namburi Sreedhar)

Received 2015-06-13

1. Introduction

The time scale calculus is a new area of mathematics that unifies and extends discrete and continuous analysis. The theory of time scales [1, 4, 5, 10] presents the tools necessary to understand and explain the mathematical structure underpinning the theories of discrete and continuous dynamic systems and allows us to connect them. It can be applied to various real life situations like epidemic models, stock markets and mathematical modeling of physical and biological systems.

Multi-point boundary value problems for ordinary differential or difference equations arise in different areas of applied mathematics and physics such as the deflection of a curved beam having a constant or varying cross section, three layer beam, electromagnetic waves or gravity driven flow and so on. For example, the vibrations of a guy wire of a uniform cross-section and composed of N parts of different densities can be set up as a multi-point boundary value problem [15] and also many problems in the theory of elastic stability can be handled as multi-point problems [19]. The study of multi-point boundary value problems for second order differential equations was introduced by Il'in and Moiseev [11, 12]. Since then, such multi-point boundary value problems (continuous or discrete cases) have been studied by many authors by using different methods such as fixed point theorems in cones.

In recent years, there is an increasing interest shown in establishing the existence of positive solutions for the iterative systems of nonlinear boundary value problems, often using Guo–Krasnosel'ski$\breve{i}$ fixed point theorem. To mention a few papers along these lines are Henderson and Ntouyas [7], Henderson, Ntouyas and Purnaras [8, 9] and Prasad, Sreedhar and Kumar [16] for ordinary differential equations and Benchohra, Henderson and Ntouyas [3], Benchohra *et al.* [2], Prasad *et al.* [17, 18] and Karaca and Tokmak [13] for dynamic equations on time scales.

Motivated by the above papers, in this paper, we are concerned with determining the eigenvalue intervals of λ_i, $1 \leq i \leq n$, for which there exist positive solutions of the iterative system of nonlinear dynamic equations on time scales,

$$\left.\begin{aligned} y_i^{\Delta\Delta}(t) + \lambda_i p_i(t) f_i(y_{i+1}(t)) = 0, \quad 1 \leq i \leq n, \quad t \in [t_1, \sigma(t_m)]_{\mathbb{T}}, \\ y_{n+1}(t) = y_1(t), \quad t \in [t_1, \sigma(t_m)]_{\mathbb{T}}, \end{aligned}\right\} \tag{1.1}$$

satisfying the m-point boundary conditions,

$$\left.\begin{aligned} &\alpha_i y_i(t_1) - \beta_i y_i^{\Delta}(t_1) = 0, \\ &\gamma_i y_i(\sigma(t_m)) + \delta_i y_i^{\Delta}(\sigma(t_m)) = \sum_{k=2}^{m-1} y_i^{\Delta}(t_k), \quad 1 \leq i \leq n, \end{aligned}\right\} \tag{1.2}$$

where T is the time scale with $t_1, \sigma^2(t_m) \in \mathbb{T}$, $0 \leq t_1 < t_2 < \cdots < t_{m-1} < \sigma(t_m)$ and $m \geq 3$, using Guo–Krasnosel'ski$\breve{i}$ fixed point theorem. By a positive solution of the boundary value problem (1.1)–(1.2), we mean an n-tuple $(y_1, y_2, \ldots, y_n) \in \left(C^2\left([t_1, \sigma(t_m)]_{\mathbb{T}}\right)\right)^n$ satisfying (1.1) and (1.2) with $y_i(t) \geq 0, i = 1, 2, \ldots, n$, for all $t \in [t_1, \sigma(t_m)]_{\mathbb{T}}$ and $(y_1, y_2, \ldots, y_n) \neq (0, 0, \ldots, 0)$.

We assume the following conditions hold throughout the paper:

(A1) $f_i : \mathbb{R}^+ \to \mathbb{R}^+$ is continuous for $1 \leq i \leq n$,

(A2) $p_i : [t_1, \sigma(t_m)]_{\mathbb{T}} \to \mathbb{R}^+$ is continuous and p_i does not vanish identically on any closed subinterval of $[t_1, \sigma(t_m)]_{\mathbb{T}}$ for $1 \leq i \leq n$,

(A3) $\alpha_i, \beta_i, \gamma_i, \delta_i$ are constants such that $\alpha_i \geq 0$, $\beta_i \geq 0$ (not simultaneously zero), $\gamma_i \geq \dfrac{\alpha_i \delta_i}{\alpha_i(t_2 - t_1) + \beta_i}$ and $\delta_i > m - 2$ for $1 \leq i \leq n$,

(A4) each of

$$f_{i0} = \lim_{x \to 0^+} \frac{f_i(x)}{x} \text{ and } f_{i\infty} = \lim_{x \to \infty} \frac{f_i(x)}{x},$$

for $1 \leq i \leq n$, exists as positive real number.

The rest of the paper is organized as follows. In Section 2, we construct the Green's function for the homogeneous problem corresponding to (1.1)–(1.2) and estimate bounds for the Green's function. In Section 3, we determine the eigenvalue intervals for which there exist positive solutions of the boundary value problem (1.1)–(1.2) by using Guo–Krasnosel'skiĭ fixed point theorem for operators on a cone in a Banach space. Finally as an application, we give an example to illustrate our result.

2. Green's function and bounds

In this section, we construct the Green's function for the homogeneous boundary value problem corresponding to (1.1)–(1.2) and estimate bounds for the Green's function.

For $1 \le i \le n$, let $G_i(t,s)$ be the Green's function for the homogeneous problem,

$$-y_i^{\Delta\Delta}(t) = 0, \quad t \in [t_1, \sigma(t_m)]_{\mathbb{T}}, \tag{2.1}$$

satisfying the boundary conditions (1.2).

Lemma 2.1. *Let $d_i = \alpha_i[\gamma_i(\sigma(t_m) - t_1) + \delta_i - m + 2] + \beta_i\gamma_i \neq 0$, $1 \le i \le n$. Then, for $1 \le i \le n$, the Green's function $G_i(t,s)$ for the homogeneous boundary value problem (2.1), (1.2) is given by*

$$G_i(t,s) = \begin{cases} G_{i_1}(t,s), & t_1 \le s \le \sigma(s) \le t_2, \\ G_{i_2}(t,s), & t_2 \le s \le \sigma(s) \le t_3, \\ \vdots & \\ G_{i_{m-1}}(t,s), & t_{m-1} \le s \le \sigma(s) \le \sigma(t_m), \end{cases} \tag{2.2}$$

where

$$G_{i_j}(t,s) = \begin{cases} \dfrac{1}{d_i}[(\alpha_i(\sigma(s) - t_1) + \beta_i)(\gamma_i(\sigma(t_m) - t) + \delta_i - m + j + 1) + (j-1)\alpha_i(t - \sigma(s))], & \sigma(s) \le t, \\ \dfrac{1}{d_i}[\alpha_i(t - t_1) + \beta_i][\gamma_i(\sigma(t_m) - \sigma(s)) + \delta_i - m + j + 1], & t \le s, \end{cases}$$

for $j = 1, 2, \ldots, m-1$.

Proof. It is easy to see that, if $h(t) \in C([t_1, \sigma(t_m)]_{\mathbb{T}}, \mathbb{R}^+)$, then the following problem,

$$-y_i^{\Delta\Delta}(t) = h(t), \quad 1 \le i \le n, \quad t \in [t_1, \sigma(t_m)]_{\mathbb{T}},$$

satisfying the boundary conditions (1.2) has a unique solution,

$$\begin{aligned} y_i(t) = {} & \frac{\beta_i}{d_i}\left[\int_{t_1}^{\sigma(t_m)} (\gamma_i(\sigma(t_m) - \sigma(s)) + \delta_i)h(s)\Delta s - \sum_{k=2}^{m-1}\int_{t_1}^{t_k} h(s)\Delta s\right] \\ & + \frac{\alpha_i}{d_i}(t - t_1)\left[\int_{t_1}^{\sigma(t_m)} (\gamma_i(\sigma(t_m) - \sigma(s)) + \delta_i)h(s)\Delta s - \sum_{k=2}^{m-1}\int_{t_1}^{t_k} h(s)\Delta s\right] - \int_{t_1}^{t}(t - \sigma(s))h(s)\Delta s. \end{aligned}$$

Rearranging the terms, it can be written as

$$\begin{aligned} y_i(t) = {} & \frac{1}{d_i}[\alpha_i(t - t_1) + \beta_i]\left[\int_{t_1}^{\sigma(t_m)} (\gamma_i(\sigma(t_m) - \sigma(s)) + \delta_i)h(s)\Delta s - \sum_{j=1}^{m-2}(m - j - 1)\int_{t_j}^{t_{j+1}} h(s)\Delta s\right] \\ & + \int_{t_1}^{t}(\sigma(s) - t)h(s)\Delta s. \end{aligned}$$

Case 1. Let $t_j \le s \le \sigma(s) \le t_{j+1}$, for $j = 1, 2, \ldots, m-2$ and $\sigma(s) \le t$. Then, for $1 \le i \le n$, we have

$$G_i(t,s) = \frac{1}{d_i}[\alpha_i(t-t_1)+\beta_i][\gamma_i(\sigma(t_m)-\sigma(s))+\delta_i-(m-j-1)]+\sigma(s)-t$$
$$= \frac{1}{d_i}[(\alpha_i(\sigma(s)-t_1)+\beta_i)(\gamma_i(\sigma(t_m)-t)+\delta_i-m+j+1)+(j-1)\alpha_i(t-\sigma(s))].$$

Case 2. Let $t_j \le s \le \sigma(s) \le t_{j+1}$, for $j=1,2,\dots,m-2$ and $t \le s$. Then, for $1 \le i \le n$, we have

$$G_i(t,s) = \frac{1}{d_i}[\alpha_i(t-t_1)+\beta_i][\gamma_i(\sigma(t_m)-\sigma(s))+\delta_i-m+j+1].$$

Case 3. Let $t_{m-1} \le s \le \sigma(s) \le \sigma(t_m)$ and $\sigma(s) \le t$. Then, for $1 \le i \le n$, we have

$$G_i(t,s) = \frac{1}{d_i}[\alpha_i(t-t_1)+\beta_i][\gamma_i(\sigma(t_m)-\sigma(s))+\delta_i]+\sigma(s)-t$$
$$= \frac{1}{d_i}[(\alpha_i(\sigma(s)-t_1)+\beta_i)(\gamma_i(\sigma(t_m)-t)+\delta_i)+(m-2)\alpha_i(t-\sigma(s))].$$

Case 4. Let $t_{m-1} \le s \le \sigma(s) \le \sigma(t_m)$ and $t \le s$. Then, for $1 \le i \le n$, we have

$$G_i(t,s) = \frac{1}{d_i}[\alpha_i(t-t_1)+\beta_i][\gamma_i(\sigma(t_m)-\sigma(s))+\delta_i].$$

□

Lemma 2.2. *Assume that the condition (A3) is satisfied. Then, for $1 \le i \le n$, the Green's function $G_i(t,s)$ of (2.1), (1.2) is positive, for all $(t,s) \in (t_1,\sigma(t_m))_{\mathbb{T}} \times (t_1,t_m)_{\mathbb{T}}$.*

Proof. By simple algebraic calculations, we can easily establish the positivity of the Green's function. □

Lemma 2.3. *Assume that the condition (A3) is satisfied. Then, for $1 \le i \le n$, the Green's function $G_i(t,s)$ in (2.2) satisfies the following inequality,*

$$G_i(t,s) \le G_i(\sigma(s),s), \ \textit{for all } (t,s) \in [t_1,\sigma(t_m)]_{\mathbb{T}} \times [t_1,t_m]_{\mathbb{T}}, \tag{2.3}$$

Proof. The Green's function $G_i(t,s)$, $1 \le i \le n$, is given in (2.2). In each case, we prove the inequality as in (2.3).

Case 1. Let $s \in [t_1,t_m]_{\mathbb{T}}$ and $\sigma(s) \le t$. Then, for $1 \le i \le n$, we have

$$\frac{G_i(t,s)}{G_i(\sigma(s),s)} = \frac{(\alpha_i(\sigma(s)-t_1)+\beta_i)(\gamma_i(\sigma(t_m)-t)+\delta_i-m+j+1)+(j-1)\alpha_i(t-\sigma(s))}{(\alpha_i(\sigma(s)-t_1)+\beta_i)(\gamma_i(\sigma(t_m)-\sigma(s))+\delta_i-m+j+1)}$$
$$\le \frac{\gamma_i(\sigma(t_m)-t)+\delta_i-m+j+1+\gamma_i(t-\sigma(s))}{\gamma_i(\sigma(t_m)-\sigma(s))+\delta_i-m+j+1} = 1.$$

Case 2. Let $s \in [t_1,t_m]_{\mathbb{T}}$ and $t \le s$. Then, for $1 \le i \le n$, we have

$$\frac{G_i(t,s)}{G_i(\sigma(s),s)} = \frac{\alpha_i(t-t_1)+\beta_i}{\alpha_i(\sigma(s)-t_1)+\beta_i}.$$

If $\alpha_i = 0$ and $\beta_i \ne 0$, then

$$\frac{G_i(t,s)}{G_i(\sigma(s),s)} = 1.$$

If $\alpha_i \ne 0$ and $\beta_i = 0$, then

$$\frac{G_i(t,s)}{G_i(\sigma(s),s)} = \frac{t-t_1}{\sigma(s)-t_1} \le 1.$$

If $\alpha_i \neq 0$ and $\beta_i \neq 0$, then

$$\frac{G_i(t,s)}{G_i(\sigma(s),s)} = \frac{\alpha_i(t-t_1)+\beta_i}{\alpha_i(\sigma(s)-t_1)+\beta_i} \leq 1.$$

Hence the result. □

Lemma 2.4. *Assume that the condition (A3) is satisfied and $s \in [t_1, t_m]_{\mathbb{T}}$. Then, for $1 \leq i \leq n$, the Green's function $G_i(t,s)$ in* (2.2) *satisfies*

$$\min_{t\in[t_{m-1},\sigma(t_m)]_{\mathbb{T}}} G_i(t,s) \geq k_i G_i(\sigma(s),s),$$

where

$$k_i = \min\left\{\frac{\delta_i - m + 2}{\gamma_i(\sigma(t_m)-t_1)+\delta_i-m+2}, \frac{\beta_i}{\alpha_i(\sigma(t_m)-t_1)+\beta_i}\right\} < 1. \tag{2.4}$$

Proof. The Green's function $G_i(t,s)$, $1 \leq i \leq n$, is given in (2.2).

Case 1. Let $t \in [t_{m-1}, \sigma(t_m)]_{\mathbb{T}}$ and $\sigma(s) \leq t$. Then, for $1 \leq i \leq n$, we have

$$\begin{aligned}\frac{G_i(t,s)}{G_i(\sigma(s),s)} &= \frac{(\alpha_i(\sigma(s)-t_1)+\beta_i)(\gamma_i(\sigma(t_m)-t)+\delta_i)+(m-2)\alpha_i(t-\sigma(s))}{(\alpha_i(\sigma(s)-t_1)+\beta_i)(\gamma_i(\sigma(t_m)-\sigma(s))+\delta_i)}\\ &\geq \frac{\delta_i-m+2}{\gamma_i(\sigma(t_m)-t_1)+\delta_i-m+2}.\end{aligned}$$

Case 2. Let $t \in [t_{m-1}, \sigma(t_m)]_{\mathbb{T}}$ and $t \leq s$. Then, for $1 \leq i \leq n$, we have

$$\frac{G_i(t,s)}{G_i(\sigma(s),s)} = \frac{\alpha_i(t-t_1)+\beta_i}{\alpha_i(\sigma(s)-t_1)+\beta_i}.$$

If $\alpha_i = 0$ and $\beta_i \neq 0$, then

$$\frac{G_i(t,s)}{G_i(\sigma(s),s)} = 1 \geq \frac{\delta_i-m+2}{\gamma_i(\sigma(t_m)-t_1)+\delta_i-m+2}.$$

If $\alpha_i \neq 0$ and $\beta_i = 0$, then

$$\frac{G_i(t,s)}{G_i(\sigma(s),s)} = \frac{t-t_1}{\sigma(s)-t_1} \geq \frac{t_2-t_1}{\sigma(t_m)-t_1} \geq \frac{\delta_i-m+2}{\gamma_i(\sigma(t_m)-t_1)+\delta_i-m+2}.$$

If $\alpha_i \neq 0$ and $\beta_i \neq 0$, then

$$\frac{G_i(t,s)}{G_i(\sigma(s),s)} = \frac{\alpha_i(t-t_1)+\beta_i}{\alpha_i(\sigma(s)-t_1)+\beta_i} \geq \frac{\beta_i}{\alpha_i(\sigma(t_m)-t_1)+\beta_i}.$$

Hence the result. □

We note that an n-tuple $(y_1(t), y_2(t), \ldots, y_n(t))$ is a solution of the boundary value problem (1.1)–(1.2) if and only if

$$y_i(t) = \lambda_i \int_{t_1}^{\sigma(t_m)} G_i(t,s)p_i(s)f_i(y_{i+1}(s))\Delta s, \quad 1 \leq i \leq n, \quad t \in [t_1, \sigma(t_m)]_{\mathbb{T}},$$

and

$$y_{n+1}(t) = y_1(t), \quad t \in [t_1, \sigma(t_m)]_{\mathbb{T}},$$

so that, in particular

$$y_1(t) = \lambda_1 \int_{t_1}^{\sigma(t_m)} G_1(t, s_1)p_1(s_1) f_1\Bigg(\lambda_2 \int_{t_1}^{\sigma(t_m)} G_2(s_1, s_2)p_2(s_2)\dots$$
$$f_{n-1}\Bigg(\lambda_n \int_{t_1}^{\sigma(t_m)} G_n(s_{n-1}, s_n)p_n(s_n) f_n(y_1(s_n))\Delta s_n\Bigg)\dots\Delta s_2\Bigg)\Delta s_1, \quad t \in [t_1, \sigma(t_m)]_{\mathbb{T}}.$$

To determine eigenvalue intervals of the boundary value problem (1.1)–(1.2), we will employ the following Guo–Krasnosel'skiĭ fixed point theorem [6, 14].

Theorem 2.5. *Let X be a Banach Space, $\kappa \subseteq X$ be a cone and suppose that Ω_1, Ω_2 are open subsets of X with $0 \in \Omega_1$ and $\overline{\Omega}_1 \subset \Omega_2$. Suppose further that $T : \kappa \cap (\overline{\Omega}_2 \backslash \Omega_1) \to \kappa$ is completely continuous operator such that either*

(i) $\|Tu\| \le \|u\|$, $u \in \kappa \cap \partial\Omega_1$ *and* $\|Tu\| \ge \|u\|$, $u \in \kappa \cap \partial\Omega_2$, *or*

(ii) $\|Tu\| \ge \|u\|$, $u \in \kappa \cap \partial\Omega_1$ *and* $\|Tu\| \le \|u\|$, $u \in \kappa \cap \partial\Omega_2$ *holds.*

Then T has a fixed point in $\kappa \cap (\overline{\Omega}_2 \backslash \Omega_1)$.

3. Positive solutions in a cone

In this section, we establish criteria to determine eigenvalue intervals for which the boundary value problem (1.1)–(1.2) has at least one positive solution in a cone.

For our construction, let $B = \{x \mid x \in C[t_1, \sigma(t_m)]_{\mathbb{T}}\}$ be a Banach space with the norm

$$\|x\| = \sup_{t \in [t_1, \sigma(t_m)]_{\mathbb{T}}} |x(t)|.$$

Define a cone $P \subset B$ by

$$P = \left\{x \in B \mid x(t) \ge 0 \text{ on } [t_1, \sigma(t_m)]_{\mathbb{T}} \text{ and } \min_{t \in [t_{m-1}, \sigma(t_m)]_{\mathbb{T}}} x(t) \ge k\|x\|\right\},$$

where

$$k = \min\{k_1, k_2, \dots, k_n\}. \tag{3.1}$$

Now, we define an integral operator $T : P \to B$, for $y_1 \in P$, by

$$\begin{aligned} Ty_1(t) = \lambda_1 \int_{t_1}^{\sigma(t_m)} G_1(t, s_1)p_1(s_1) f_1\Bigg(\lambda_2 \int_{t_1}^{\sigma(t_m)} G_2(s_1, s_2)p_2(s_2)\dots \\ f_{n-1}\Bigg(\lambda_n \int_{t_1}^{\sigma(t_m)} G_n(s_{n-1}, s_n)p_n(s_n) f_n(y_1(s_n))\Delta s_n\Bigg)\dots\Delta s_2\Bigg)\Delta s_1. \end{aligned} \tag{3.2}$$

Notice from (A1), (A2) and Lemma 2.2 that, for $y_1 \in P$, $Ty_1(t) \ge 0$ on $[t_1, \sigma(t_m)]_{\mathbb{T}}$. Also, for $y_1 \in P$, we have from Lemma 2.3, that

$$Ty_1(t) \le \lambda_1 \int_{t_1}^{\sigma(t_m)} G_1(\sigma(s_1), s_1)p_1(s_1) f_1\Bigg(\lambda_2 \int_{t_1}^{\sigma(t_m)} G_2(s_1, s_2)p_2(s_2)\dots$$
$$f_{n-1}\Bigg(\lambda_n \int_{t_1}^{\sigma(t_m)} G_n(s_{n-1}, s_n)p_n(s_n) f_n(y_1(s_n))\Delta s_n\Bigg)\dots\Delta s_2\Bigg)\Delta s_1$$

so that

$$\|Ty_1\| \le \lambda_1 \int_{t_1}^{\sigma(t_m)} G_1(\sigma(s_1),s_1)p_1(s_1)f_1\Bigg(\lambda_2 \int_{t_1}^{\sigma(t_m)} G_2(s_1,s_2)p_2(s_2)\dots \\ f_{n-1}\Bigg(\lambda_n \int_{t_1}^{\sigma(t_m)} G_n(s_{n-1},s_n)p_n(s_n)f_n(y_1(s_n))\Delta s_n\Bigg)\dots\Delta s_2\Bigg)\Delta s_1. \tag{3.3}$$

Next, if $y_1 \in P$, we have from Lemma 2.4, (3.1) and (3.3) that

$$\begin{aligned}\min_{t\in[t_{m-1},\sigma(t_m)]_{\mathbb{T}}} Ty_1(t) &= \min_{t\in[t_{m-1},\sigma(t_m)]_{\mathbb{T}}} \Bigg\{\lambda_1 \int_{t_1}^{\sigma(t_m)} G_1(t,s_1)p_1(s_1)f_1\Bigg(\lambda_2 \int_{t_1}^{\sigma(t_m)} G_2(s_1,s_2)p_2(s_2)\dots \\ &\quad f_{n-1}\Bigg(\lambda_n \int_{t_1}^{\sigma(t_m)} G_n(s_{n-1},s_n)p_n(s_n)f_n(y_1(s_n))\Delta s_n\Bigg)\dots\Delta s_2\Bigg)\Delta s_1\Bigg\} \\ &\ge \lambda_1 k \int_{t_1}^{\sigma(t_m)} G_1(\sigma(s_1),s_1)p_1(s_1)f_1\Bigg(\lambda_2 \int_{t_1}^{\sigma(t_m)} G_2(s_1,s_2)p_2(s_2)\dots \\ &\quad f_{n-1}\Bigg(\lambda_n \int_{t_1}^{\sigma(t_m)} G_n(s_{n-1},s_n)p_n(s_n)f_n(y_1(s_n))\Delta s_n\Bigg)\dots\Delta s_2\Bigg)\Delta s_1 \\ &\ge k\|Ty_1\|.\end{aligned}$$

Hence, $Ty_1 \in P$ and so $T : P \to P$. Further, the operator T is completely continuous operator by an application of the Ascoli-Arzela Theorem.

Now, we seek suitable fixed points of T belonging to the cone P. For our first result, define positive numbers M_1 and M_2 by

$$M_1 = \max_{1\le i\le n}\left\{\left[k^2 \int_{t_{m-1}}^{\sigma(t_m)} G_i(\sigma(s),s)p_i(s)\Delta s f_{i\infty}\right]^{-1}\right\}$$

and

$$M_2 = \min_{1\le i\le n}\left\{\left[\int_{t_1}^{\sigma(t_m)} G_i(\sigma(s),s)p_i(s)\Delta s f_{i0}\right]^{-1}\right\}.$$

Theorem 3.1. *Assume that the conditions (A1)–(A4) are satisfied. Then, for each $\lambda_1, \lambda_2, \dots, \lambda_n$ satisfying*

$$M_1 < \lambda_i < M_2, \quad 1 \le i \le n, \tag{3.4}$$

there exists a positive solution $(y_1, y_2, \dots, y_n)$ satisfying (1.1)–(1.2) such that $y_i(t) > 0$, $1 \le i \le n$, on $(t_1, \sigma(t_m))_{\mathbb{T}}$.

Proof. Let λ_j, $1 \le j \le n$, be given as in (3.4). Now, let $\epsilon > 0$ be chosen such that

$$\max_{1\le i\le n}\left\{\left[k^2 \int_{t_{m-1}}^{\sigma(t_m)} G_i(\sigma(s),s)p_i(s)\Delta s(f_{i\infty}-\epsilon)\right]^{-1}\right\} \le \min_{1\le j\le n}\lambda_j,$$

and

$$\max_{1\le j\le n}\lambda_j \le \min_{1\le i\le n}\left\{\left[\int_{t_1}^{\sigma(t_m)} G_i(\sigma(s),s)p_i(s)\Delta s(f_{i0}+\epsilon)\right]^{-1}\right\}.$$

We seek fixed points of the completely continuous operator $T : P \to P$ defined by (3.2). Now, from the definitions of f_{i0}, $1 \le i \le n$, there exists an $H_1 > 0$ such that, for each $1 \le i \le n$,

$$f_i(x) \leq (f_{i0} + \epsilon)x, \quad 0 < x \leq H_1.$$

Let $y_1 \in P$ with $\|y_1\| = H_1$. We first have from Lemma 2.3 and the choice of ϵ, for $t_1 \leq s_{n-1} \leq \sigma(t_m)$,

$$\begin{aligned}\lambda_n \int_{t_1}^{\sigma(t_m)} G_n(s_{n-1}, s_n)p_n(s_n)f_n(y_1(s_n))\Delta s_n &\leq \lambda_n \int_{t_1}^{\sigma(t_m)} G_n(\sigma(s_n), s_n)p_n(s_n)f_n(y_1(s_n))\Delta s_n \\ &\leq \lambda_n \int_{t_1}^{\sigma(t_m)} G_n(\sigma(s_n), s_n)p_n(s_n)(f_{n0} + \epsilon)y_1(s_n)\Delta s_n \\ &\leq \lambda_n \int_{t_1}^{\sigma(t_m)} G_n(\sigma(s_n), s_n)p_n(s_n)\Delta s_n(f_{n0} + \epsilon)\|y_1\| \\ &\leq \|y_1\| = H_1.\end{aligned}$$

It follows in a similar manner from Lemma 2.3 and the choice of ϵ that, for $t_1 \leq s_{n-2} \leq \sigma(t_m)$,

$$\begin{aligned}&\lambda_{n-1} \int_{t_1}^{\sigma(t_m)} G_{n-1}(s_{n-2}, s_{n-1})p_{n-1}(s_{n-1})f_{n-1}\left(\lambda_n \int_{t_1}^{\sigma(t_m)} G_n(s_{n-1}, s_n)p_n(s_n)f_n(y_1(s_n))\Delta s_n\right)\Delta s_{n-1} \\ &\leq \lambda_{n-1} \int_{t_1}^{\sigma(t_m)} G_{n-1}(\sigma(s_{n-1}), s_{n-1})p_{n-1}(s_{n-1})\Delta s_{n-1}(f_{n-1,0} + \epsilon)H_1 \\ &\leq H_1.\end{aligned}$$

Continuing with this bootstrapping argument, we have, for $t_1 \leq t \leq \sigma(t_m)$,

$$\lambda_1 \int_{t_1}^{\sigma(t_m)} G_1(t, s_1)p_1(s_1)f_1\left(\lambda_2 \int_{t_1}^{\sigma(t_m)} G_2(s_1, s_2)p_2(s_2)\ldots f_n(y_1(s_n))\Delta s_n \ldots \Delta s_2\right)\Delta s_1 \leq H_1,$$

so that, for $t_1 \leq t \leq \sigma(t_m)$,

$$Ty_1(t) \leq H_1.$$

Hence, $\|Ty_1\| \leq H_1 = \|y_1\|$. If we set

$$\Omega_1 = \{x \in B \mid \|x\| < H_1\},$$

then

$$\|Ty_1\| \leq \|y_1\|, \quad \text{for} \quad y_1 \in P \cap \partial\Omega_1. \tag{3.5}$$

Next, from the definitions of $f_{i\infty}$, $1 \leq i \leq n$, there exists $\overline{H}_2 > 0$ such that, for each $1 \leq i \leq n$,

$$f_i(x) \geq (f_{i\infty} - \epsilon)x, \quad x \geq \overline{H}_2.$$

Let

$$H_2 = \max\left\{2H_1, \frac{\overline{H}_2}{k}\right\}.$$

Choose $y_1 \in P$ and $\|y_1\| = H_2$. Then,

$$\min_{t \in [t_{m-1}, \sigma(t_m)]_{\mathbb{T}}} y_1(t) \geq k\|y_1\| \geq \overline{H}_2.$$

From Lemma 2.4, (3.1) and choice of ϵ, for $t_1 \leq s_{n-1} \leq \sigma(t_m)$, we have that

$$\begin{aligned}\lambda_n \int_{t_1}^{\sigma(t_m)} G_n(s_{n-1}, s_n)p_n(s_n)f_n(y_1(s_n))\Delta s_n &\geq \lambda_n \int_{t_{m-1}}^{\sigma(t_m)} G_n(s_{n-1}, s_n)p_n(s_n)f_n(y_1(s_n))\Delta s_n \\ &\geq k\lambda_n \int_{t_{m-1}}^{\sigma(t_m)} G_n(\sigma(s_n), s_n)p_n(s_n)(f_{n\infty} - \epsilon)y_1(s_n)\Delta s_n\end{aligned}$$

$$\begin{aligned}&\geq k^2\lambda_n \int_{t_{m-1}}^{\sigma(t_m)} G_n(\sigma(s_n), s_n)p_n(s_n)\Delta s_n (f_{n\infty}-\epsilon)\|y_1\|\\ &\geq \|y_1\| = H_2.\end{aligned}$$

It follows in a similar manner from Lemma 2.4, (3.1) and choice of ϵ, for $t_1 \leq s_{n-2} \leq \sigma(t_m)$,

$$\begin{aligned}&\lambda_{n-1}\int_{t_1}^{\sigma(t_m)} G_{n-1}(s_{n-2}, s_{n-1})p_{n-1}(s_{n-1})f_{n-1}\left(\lambda_n \int_{t_1}^{\sigma(t_m)} G_n(s_{n-1}, s_n)p_n(s_n)f_n(y_1(s_n))\Delta s_n\right)\Delta s_{n-1}\\ &\geq k\lambda_{n-1}\int_{t_{m-1}}^{\sigma(t_m)} G_{n-1}(\sigma(s_{n-1}), s_{n-1})p_{n-1}(s_{n-1})\Delta s_{n-1}(f_{n-1,\infty}-\epsilon)H_2\\ &\geq k^2\lambda_{n-1}\int_{t_{m-1}}^{\sigma(t_m)} G_{n-1}(\sigma(s_{n-1}), s_{n-1})p_{n-1}(s_{n-1})\Delta s_{n-1}(f_{n-1,\infty}-\epsilon)H_2\\ &\geq H_2.\end{aligned}$$

Again, using a bootstrapping argument, we have

$$\lambda_1 \int_{t_1}^{\sigma(t_m)} G_1(t, s_1)p_1(s_1)f_1\left(\lambda_2 \int_{t_1}^{\sigma(t_m)} G_2(s_1, s_2)p_2(s_2)\ldots f_n(y_1(s_n))\Delta s_n \ldots \Delta s_2\right)\Delta s_1 \geq H_2,$$

so that

$$Ty_1(t) \geq H_2 = \|y_1\|.$$

Hence, $\|Ty_1\| \geq \|y_1\|$. So if we set

$$\Omega_2 = \{x \in B \mid \|x\| < H_2\},$$

then

$$\|Ty_1\| \geq \|y_1\|, \quad \text{for} \quad y_1 \in P \cap \partial\Omega_2. \tag{3.6}$$

Applying Theorem 2.5 to (3.5) and (3.6), we obtain that T has a fixed point $y_1 \in P \cap (\overline{\Omega}_2 \backslash \Omega_1)$. As such, setting $y_{n+1} = y_1$, we obtain a positive solution $(y_1, y_2, \ldots, y_n)$ of (1.1)–(1.2) given iteratively by

$$y_j(t) = \lambda_j \int_{t_1}^{\sigma(t_m)} G_j(t, s)p_j(s)f_j(y_{j+1}(s))\Delta s, \qquad j = n, n-1, \ldots, 1.$$

The proof is completed. □

Prior to our next result, we define the positive numbers M_3 and M_4 by

$$M_3 = \max_{1\leq i\leq n}\left\{\left[k^2\int_{t_{m-1}}^{\sigma(t_m)} G_i(\sigma(s), s)p_i(s)\Delta s f_{i0}\right]^{-1}\right\}$$

and

$$M_4 = \min_{1\leq i\leq n}\left\{\left[\int_{t_1}^{\sigma(t_m)} G_i(\sigma(s), s)p_i(s)\Delta s f_{i\infty}\right]^{-1}\right\}.$$

Theorem 3.2. *Assume that the conditions (A1)–(A4) are satisfied. Then, for each $\lambda_1, \lambda_2, \ldots, \lambda_n$ satisfying*

$$M_3 < \lambda_i < M_4, \quad 1 \leq i \leq n, \tag{3.7}$$

there exists a positive solution $(y_1, y_2, \ldots, y_n)$ satisfying (1.1)–(1.2) such that $y_i(t) > 0$, $1 \leq i \leq n$ on $(t_1, \sigma(t_m))_{\mathbb{T}}$.

Proof. Let λ_j, $1 \leq j \leq n$ be given as in (3.7). Now, let $\epsilon > 0$ be chosen such that

$$\max_{1\le i\le n}\left\{\left[k^2\int_{t_{m-1}}^{\sigma(t_m)} G_i(\sigma(s),s)p_i(s)\Delta s(f_{i0}-\epsilon)\right]^{-1}\right\}\le \min_{1\le j\le n}\lambda_j,$$

and

$$\max_{1\le j\le n}\lambda_j\le \min_{1\le i\le n}\left\{\left[\int_{t_1}^{\sigma(t_m)} G_i(\sigma(s),s)p_i(s)\Delta s(f_{i\infty}+\epsilon)\right]^{-1}\right\}.$$

Let T be the cone preserving, completely continuous operator that was defined by (3.2). From the definitions of f_{i0}, $1\le i\le n$, there exists $\overline{H}_3>0$ such that, for each $1\le i\le n$,

$$f_i(x)\ge (f_{i0}-\epsilon)x,\quad 0<x\le \overline{H}_3.$$

Also, from the definitions of f_{i0}, it follows that $f_{i0}(0)=0,\ \ 1\le i\le n$, and so there exist $0<K_n<K_{n-1}<\ldots<K_2<\overline{H}_3$ such that

$$\lambda_i f_i(x)\le \frac{K_{i-1}}{\displaystyle\int_{t_1}^{\sigma(t_m)} G_i(\sigma(s),s)p_i(s)\Delta s},\quad t\in[0,K_i],\quad 3\le i\le n,$$

and

$$\lambda_2 f_2(x)\le \frac{\overline{H}_3}{\displaystyle\int_{t_1}^{\sigma(t_m)} G_2(\sigma(s),s)p_2(s)\Delta s},\quad t\in[0,K_2].$$

Choose $y_1\in P$ with $\|y_1\|=K_n$. Then, we have

$$\begin{aligned}\lambda_n\int_{t_1}^{\sigma(t_m)} G_n(s_{n-1},s_n)p_n(s_n)f_n(y_1(s_n))\Delta s_n &\le \lambda_n\int_{t_1}^{\sigma(t_m)} G_n(\sigma(s_n),s_n)p_n(s_n)f_n(y_1(s_n))\Delta s_n\\ &\le \frac{\displaystyle\int_{t_1}^{\sigma(t_m)} G_n(\sigma(s_n),s_n)p_n(s_n)K_{n-1}\Delta s_n}{\displaystyle\int_{t_1}^{\sigma(t_m)} G_n(\sigma(s_n),s_n)p_n(s_n)\Delta s_n}\\ &=K_{n-1}.\end{aligned}$$

Continuing with this bootstrapping argument, it follows that

$$\lambda_2\int_{t_1}^{\sigma(t_m)} G_1(s_1,s_2)p_2(s_2)f_2\left(\lambda_3\int_{t_1}^{\sigma(t_m)} G_2(s_2,s_3)p_3(s_3)\ldots f_n(y_1(s_n))\Delta s_n\ldots\Delta s_3\right)\Delta s_2\le \overline{H}_3.$$

Then,

$$\begin{aligned}Ty_1(t)&=\lambda_1\int_{t_1}^{\sigma(t_m)} G_1(t,s_1)p_1(s_1)f_1\left(\lambda_2\int_{t_1}^{\sigma(t_m)} G_2(s_1,s_2)p_2(s_2)\ldots f_n(y_1(s_n))\Delta s_n\ldots\Delta s_2\right)\Delta s_1\\ &\ge k^2\lambda_1\int_{t_{m-1}}^{\sigma(t_m)} G_1(\sigma(s_1),s_1)p_1(s_1)(f_{1,0}-\epsilon)\|y_1\|\Delta s_1\\ &\ge \|y_1\|.\end{aligned}$$

So, $\|Ty_1\|\ge\|y_1\|$. If we put

$$\Omega_3=\{x\in B\mid \|x\|<K_n\},$$

then

$$\|Ty_1\| \geq \|y_1\|, \quad \text{for} \quad y_1 \in P \cap \partial\Omega_3. \tag{3.8}$$

Since each $f_{i\infty}$ is assumed to be a positive real number, it follows that f_i, $1 \leq i \leq n$, is unbounded at ∞. For each $1 \leq i \leq n$, set

$$f_i^*(x) = \sup_{0 \leq s \leq x} f_i(s).$$

Then, it is straightforward that, for each $1 \leq i \leq n$, f_i^* is a nondecreasing real-valued function, $f_i \leq f_i^*$ and

$$\lim_{x \to \infty} \frac{f_i^*(x)}{x} = f_{i\infty}.$$

Next, by definition of $f_{i\infty}$, $1 \leq i \leq n$, there exists $\overline{H}_4 > 0$ such that, for each $1 \leq i \leq n$,

$$f_i^*(x) \leq (f_{i\infty} + \epsilon)x, \quad x \geq \overline{H}_4.$$

Then, for $H_4 = \max\{2\overline{H}_3, \overline{H}_4\}$, and for each $1 \leq i \leq n$, we have

$$f_i^*(x) \leq f_i^*(H_4), \quad 0 < x \leq H_4.$$

Choose $y_1 \in P$ with $\|y_1\| = H_4$. Then, using the usual bootstrapping argument, we have

$$\begin{aligned}
Ty_1(t) &= \lambda_1 \int_{t_1}^{\sigma(t_m)} G_1(t, s_1) p_1(s_1) f_1(\lambda_2 \ldots) \Delta s_1 \\
&\leq \lambda_1 \int_{t_1}^{\sigma(t_m)} G_1(t, s_1) p_1(s_1) f_1^*(\lambda_2 \ldots) \Delta s_1 \\
&\leq \lambda_1 \int_{t_1}^{\sigma(t_m)} G_1(\sigma(s_1), s_1) p_1(s_1) f_1^*(H_4) \Delta s_1 \\
&\leq \lambda_1 \int_{t_1}^{\sigma(t_m)} G_1(\sigma(s_1), s_1) p_1(s_1) \Delta s_1 (f_{1\infty} + \epsilon) H_4 \\
&\leq H_4 = \|y_1\|,
\end{aligned}$$

Hence, $\|Ty_1\| \leq \|y_1\|$. So, if we let

$$\Omega_4 = \{x \in B \mid \|x\| < H_4\},$$

then

$$\|Ty_1\| \leq \|y_1\|, \quad \text{for} \quad y_1 \in P \cap \partial\Omega_4. \tag{3.9}$$

Applying Theorem 2.5 to (3.8) and (3.9), we obtain that T has a fixed point $y_1 \in P \cap (\overline{\Omega}_4 \backslash \Omega_3)$, which in turn with $y_{n+1} = y_1$, yields an n-tuple $(y_1, y_2, \ldots, y_n)$ satisfying (1.1)–(1.2) for the chosen values of λ_i, $1 \leq i \leq n$. The proof is completed. □

4. Example

Let us introduce an example to illustrate the above result. Let $\mathbb{T} = \{(\frac{1}{2})^p : p \in \mathbb{N}_0\} \cup [1, 2]$. Now, consider the following boundary value problem on time scales,

$$\left.\begin{aligned}
y_1^{\Delta\Delta}(t) + \lambda_1 p_1(t) f_1(y_2(t)) &= 0, \quad t \in \left[\frac{1}{2}, \sigma(2)\right]_{\mathbb{T}}, \\
y_2^{\Delta\Delta}(t) + \lambda_2 p_2(t) f_2(y_3(t)) &= 0, \quad t \in \left[\frac{1}{2}, \sigma(2)\right]_{\mathbb{T}}, \\
y_3^{\Delta\Delta}(t) + \lambda_3 p_3(t) f_3(y_1(t)) &= 0, \quad t \in \left[\frac{1}{2}, \sigma(2)\right]_{\mathbb{T}},
\end{aligned}\right\} \tag{4.1}$$

$$\left.\begin{aligned} 2y_1\left(\frac{1}{2}\right) - 5y_1^{\Delta}\left(\frac{1}{2}\right) = 0,\ 4y_1(\sigma(2)) + 3y_1^{\Delta}(\sigma(2)) = y_1^{\Delta}(1),\\ y_2\left(\frac{1}{2}\right) - 2y_2^{\Delta}\left(\frac{1}{2}\right) = 0,\ 3y_2(\sigma(2)) + 4y_2^{\Delta}(\sigma(2)) = y_2^{\Delta}(1),\\ 3y_3\left(\frac{1}{2}\right) - y_3^{\Delta}\left(\frac{1}{2}\right) = 0,\ 5y_3(\sigma(2)) + 3y_3^{\Delta}(\sigma(2)) = y_3^{\Delta}(1),\end{aligned}\right\} \tag{4.2}$$

where

$$\begin{aligned} f_1(y_2) &= \frac{38}{5}\left|sin\left(\frac{y_2}{2}\right)\right| + 750000y_2e^{-\frac{1}{y_2^2}},\\ f_2(y_3) &= \frac{7}{2}\left|sin(y_3)\right| + 600000y_3e^{-\frac{1}{y_3^4}},\\ f_3(y_1) &= \frac{9}{2}\left|sin\left(\frac{y_1}{2}\right)\right| + 540000y_1e^{-\frac{1}{y_1^3}},\end{aligned}$$

and $p_1(t) = p_2(t) = p_3(t) = 1$.
The Green's function $G_i(t,s)$ for $i = 1,2,3$, in Lemma 2.1 is

$$G_i(t,s) = \begin{cases} G_{i_1}(t,s), & \frac{1}{2} \le s \le \sigma(s) \le 1,\\ G_{i_2}(t,s), & 1 \le s \le \sigma(s) \le \sigma(2),\end{cases}$$

where

$$G_{1_1}(t,s) = \begin{cases} \frac{1}{36}(2\sigma(s)+4)[4(\sigma(2)-t)+2], & \sigma(s) \le t,\\ \frac{1}{36}(2t+4)[4(\sigma(2)-\sigma(s))+2], & t \le s,\end{cases}$$

$$G_{1_2}(t,s) = \begin{cases} \frac{1}{36}[(2\sigma(s)+4)(4(\sigma(2)-t)+3)+2(t-\sigma(s))], & \sigma(s) \le t,\\ \frac{1}{36}(2t+4)[4(\sigma(2)-\sigma(s))+3], & t \le s,\end{cases}$$

$$G_{2_1}(t,s) = \begin{cases} \frac{2}{27}\left(\sigma(s)+\frac{3}{2}\right)[3(\sigma(2)-t)+3], & \sigma(s) \le t,\\ \frac{2}{27}\left(t+\frac{3}{2}\right)[3(\sigma(2)-\sigma(s))+3], & t \le s,\end{cases}$$

$$G_{2_2}(t,s) = \begin{cases} \frac{2}{27}\left[\left(\sigma(s)+\frac{3}{2}\right)[3(\sigma(2)-t)+4]+t-\sigma(s)\right], & \sigma(s) \le t,\\ \frac{2}{27}\left(t+\frac{3}{2}\right)[3(\sigma(2)-\sigma(s))+4], & t \le s,\end{cases}$$

and

$$G_{3_1}(t,s) = \begin{cases} \frac{2}{67}\left(3\sigma(s)-\frac{1}{2}\right)[5(\sigma(2)-t)+2], & \sigma(s) \le t,\\ \frac{2}{67}\left(3t-\frac{1}{2}\right)[5(\sigma(2)-\sigma(s))+2], & t \le s,\end{cases}$$

$$G_{3_2}(t,s) = \begin{cases} \frac{2}{67}\left[\left(3\sigma(s)-\frac{1}{2}\right)[5(\sigma(2)-t)+3]+3(t-\sigma(s))\right], & \sigma(s) \le t,\\ \frac{2}{67}\left(3t-\frac{1}{2}\right)[5(\sigma(2)-\sigma(s))+3], & t \le s.\end{cases}$$

Clearly, the Green functions $G_1(t,s)$, $G_2(t,s)$ and $G_3(t,s)$ are positive. By algebraic calculations, we get

$$k = 0.1818181818, f_{10} = 3.8, f_{20} = 3.5, f_{30} = 2.25, f_{1\infty} = 750000, f_{2\infty} = 600000, f_{3\infty} = 540000,$$

$$M_1 = \max\{0.00004229126214, 0.00004188461538, 0.00009043953592\},$$

and

$$M_2 = \min\{0.1346969319, 0.1234285714, 0.3892519971\}.$$

Employing Theorem 3.1, we get an eigenvalue interval $0.00009043953592 < \lambda_i < 0.1234285714$, $i = 1, 2, 3$, for which the boundary value problem (4.1)–(4.2) has a positive solution.

Acknowledgements:

The authors thank the referees for their valuable suggestions.

References

[1] R. P. Agarwal, M. Bohner, *Basic calculus on time scales and some of its applications*, Results Math., **35** (1999), 3–22. 1
[2] M. Benchohra, F. Berhoun, S. Hamani, J. Henderson, S. K. Ntouyas, A. Ouahab, I. K. Purnaras, *Eigenvalues for iterative systems of nonlinear boundary value problems on time scales*, Nonlinear Dyn. Syst. Theory, **9** (2009), 11–22. 1
[3] M. Benchohra, J. Henderson, S. K. Ntouyas, *Eigenvalue problems for systems of nonlinear boundary value problems on time scales*, Adv. Difference Equ., **2007** (2007), 10 pages. 1
[4] M. Bohner, A. Peterson, *Dynamic equations on time scales, an introduction with applications*, Birkhäuser, Boston, MA, (2001). 1
[5] M. Bohner, A. Peterson, *Advances in dynamic equations on time scales*, Birkhäuser Boston, Inc., Boston, MA, (2003). 1
[6] D. J. Guo, V. Lakshmikantham, *Nonlinear problems in abstract cones*, Academic Press, Inc., Boston, MA, (1988). 2
[7] J. Henderson, S. K. Ntouyas, *Positive solutions for systems of nth order three-point nonlocal boundary value problems*, Electron. J. Qual. Theory Differ. Equ., **2007** (2007), 12 pages. 1
[8] J. Henderson, S. K. Ntouyas, I. K. Purnaras, *Positive solutions for systems of generalized three-point nonlinear boundary value problems*, Comment. Math. Univ. Carolin., **49** (2008), 79–91. 1
[9] J. Henderson, S. K. Ntouyas, I. K. Purnaras, *Positive solutions for systems of second order four-point nonlinear boundary value problems*, Commun. Appl. Anal., **12** (2008), 29–40. 1
[10] S. Hilger, *Analysis on measure chains-a unified approach to continuous and discrete calculus*, Results Math., **18** (1990), 18–56. 1
[11] V. A. Il'in, E. I. Moiseev, *Nonlocal boundary-value problem of the first kind for a Sturm-Liouville operator in its differential and finite difference aspects*, Differ. Equ., **23** (1987), 803–810. 1
[12] V. A. Il'in, E. I. Moiseev, *Nonlocal boundary value problem of the second kind for a Sturm-Liouville operator*, Differ. Equ., **23** (1987), 979–987. 1
[13] I. Y. Karaca, F. Tokmak, *Eigenvalues for iterative systems of nonlinear m-point boundary value problems on time scales*, Bound. Value Probl., **2014** (2014), 17 pages. 1
[14] M. A. Krasnosel'skiĭ, *Positive solutions of operator equations*, P. Noordhoff Ltd., Groningen, The Netherlands, (1964). 2
[15] M. Moshinsky, *Sobre los problemas de condiciones a la frontiera en una dimension de caracteristicas discontinuas*, Bol. Soc. Mat. Mexicana, **7** (1950), 10–25. 1
[16] K. R. Prasad, N. Sreedhar, K. R. Kumar, *Solvability of iterative systems of three-point boundary value problems*, TWMS J. Appl. Eng. Math., **3** (2013), 147–159. 1
[17] K. R. Prasad, N. Sreedhar, Y. Narasimhulu, *Eigenvalue intervals for iterative systems of nonlinear m-point boundary value problems on time scales*, Differ. Equ. Dyn. Syst., **22** (2014), 353–368. 1
[18] K. R. Prasad, N. Sreedhar, M. A. S. Srinivas, *Eigenvalue intervals for iterative systems of nonlinear three-point boundary value problems on time scales*, Nonlinear Stud., **22** (2015), 419–431. 1
[19] S. P. Timoshenko, *Theory of elastic stability*, McGraw-Hill Book Co., New York-Toronto-London, (1961). 1